String theory
Profielwerkstuk

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Inhoudsopgave

Head-question and sub-questions

Head-question: To what extent is string theory related to dimensions and mathematics?

Sub-questions:

1. What is string theory?
2. What are dimensions?
3. In what way does the z-axis help to illustrate the concept of additional dimensions?
4. What is the mathematics and basic physics of the string theory and its dimensions?
5. Where are these additional dimensions?
6. How does the frequency of a vibrating string change when adjusting the tension, thickness, or length, and how can these observations help us understand vibrations on a quantum level in string theory

Hypothesis

Although we do know that dimensions play a crucial role in string theory we did not know in what extent it has to do with the math's and the 10 dimensions of the string theory. Though we are uncertain about the extent to which matrices are involved. The mathematical framework of dimensions is fundamental in elucidating their significance in string theory. Specifically, the multi-dimensional nature of strings necessitates a sophisticated understanding of higher-dimensional spaces, which is often facilitated by the use of advanced mathematical tools, including matrices. So exploring the mathematics of dimensions not only highlights their importance in string theory but also underscores the potential involvement of matrices in describing these complex structures. We know that all theories of the string theory use supersymmetry, and to explain the math's we will be using differential geometry the most. This branch of mathematics deals with shapes and the properties of curves and surfaces. In the string theory, differential geometry is essential for describing the geometry of the higher-dimensional spaces in which strings propagate.

Procedure

Our profile project aims to explore string theory, a complex and intriguing area of theoretical physics that proposes that the fundamental particles of the universe are not point-like dots, but rather, vibrating strings. We are continuing Fitou's research on black and white holes with this project using it as a point of inspiration and help. This project will involve a combination of independent research, Fitou teaching Merel stuff and the other way around, and an expert interview to provide a thorough understanding of String Theory and its implications in the scientific community. We will engage in several activities, including individual study, educational sessions which we just watch movies or read books about it. The ultimate goal is to not only grasp the basic principles of String Theory but also to communicate them in a clear and accessible way.

The first step in our project involves an in-depth initial research phase, where we will independently explore the fundamental concepts of String Theory. Fitou has had an obsession with this subject for a long time so she know a lot about it that's why she will go more into depth while Merel will study by herself in accessible resources, such as educational videos, articles, and documentaries that are designed to explain the basics of string theory in a way that is easy to understand. She will focus on resources like PBS space Time, which offers a series of videos that break down complex physics topics, as well as Kurzgesagt's animations, which present scientific ideas in a visually engaging manner. The goal of this phase is for Merel to build a foundational understanding of String Theory, including concepts such as the nature of strings, the role of dimensions beyond the familiar three, and the ways in which String Theory attempts to reconcile the seemingly incompatible worlds of quantum mechanics and general relativity.

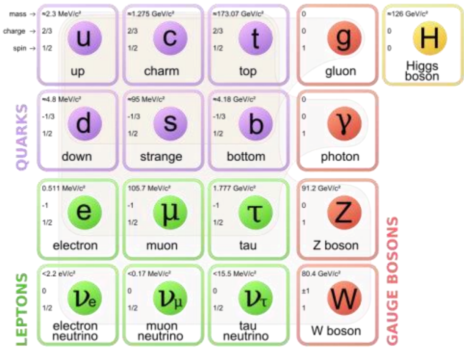
Theoretic stats

Let’s begin with the very basics, instead of viewing particles as zero-dimensional points, the string theory posits that the basic building blocks of the universe are one-dimensional "strings" that vibrate at specific frequencies. These vibrations correspond to different particles, as an example, one mode of vibration might produce an electron, while another might produce a photon. That said the purpose of string theory is to provide a unified framework that explains all fundamental forces and particles in the universe, including gravity, within a single theoretical structure.

It is something that has to reconcile quantum mechanics, which describes the behavior of particles at small scales with general relativity, which explains gravity and spacetime on large scales. Two theories that are currently incompatible in extreme conditions like black holes or the Big Bang. String theory aims to unify the four fundamental forces : gravity, electromagnetism, the strong nuclear force, and the weak nuclear force. By describing all particles as different vibrations of tiny, one-dimensional strings. This theory overcomes limitations in both the *Standard Model* of particle physics, which doesn’t account for gravity, and general relativity, which breaks down at quantum scales. Ultimately, string theory is the "Theory of Everything" that explains the workings of the universe from the smallest particles to the largest cosmic structures!!! (string theory, sd) (string theory pdf) (string lectures, sd) (standart model, sd) (standart model, 2024)

What it matter?

‘’So our universe is made out of matter.. but what does that mean?’’. When we take a closer look at matter we will see that it is made out of particles there are different types of particles: electrons, quarks or neutrinos. Particles can interact with one another by exchanging other particles. All these different particles are accounted for, what we call the standard model. The standard model is a very precise mathematical description of the quantum world it contains. It also covers the forces that govern how particles movee, interact and bind together to give a shape to the world around us.
But how does it really work?



Let’s zoom in for instance on a pen, when we zoom in enough we see molecules that are made up of atoms bound together. The electrons in atoms are the first to be put in the standard model. Electrons are bound to an atoms nucleus by electro magnetism. They attract each other by exchanging something called ‘’photons’’(y in the standard model) these are one of the fundamental forces in the standard model. The nucleus contains protons and neutrons, these are made up of quarks these are invisible. A proton has 2 ‘’up’’ quarks and one ‘’down’’ quark and a neutrino has the opposite. The nucleus is held together by the strong force (g in the standard model) gluons carry the strong force (hence the g) electrons alongside up and down quarks is really all we need to explain normal matter, however there are actually six quarks: down and up, strange and charm, and bottom and top. Leptons are another type of elementary particle. They are not made up of smaller particles. Leptons, along with quarks, form the basic building blocks of matter in the universe. The same goes for electrons, they also have other heavier versions called ‘’muons’’ and ‘’tau’’. Then there are 2 things called ‘’W’’ and ‘’Z’’ these 2 unlike the photons have a mass, these carry the weak force and thus the final force of the standard model. But there is another type of particle in the standard model called neutrinos. And then we have Higgs boson, this is a quantum ripple in the background energy field of the universe often called nothing.

The Standard Model is built on the principles of *gauge theory*, specifically using the symmetry group SU(3), SU(2), U(1) Each of these groups corresponds to a fundamental interaction:

SU(3): Strong force, which acts on quarks.

SU(2): Weak force, responsible for processes like radioactive decay.

U(1): Electromagnetic force, acting on electrically charged particles.

(standart model, sd) (standart model, 2024) (hartnett, 2021) (standartmodel summary, 1998) (Ling-Fong) (Horejsi)

Supersymmetry (susy)

Supersymmetry is a theoretical concept in physics suggesting that every fundamental particle in the standard model we know has a partner particle, known as a "super partner," which has different quantum properties. Think of these partners as mirrored versions of the familiar particles, but they haven't been observed yet. The idea behind supersymmetry is to address certain gaps in our current understanding, particularly in the Standard Model of particle physics, by balancing forces and resolving some inconsistencies. For example, SUSY could help explain dark matter or why particles have the masses they do, potentially pointing us toward a deeper underlying symmetry in nature. While experimental searches haven't yet confirmed supersymmetry, scientists still find it compelling as it could unlock a more complete picture of the universe.

So in string theory, supersymmetry plays a key role because it allows the theory to be mathematically consistent and potentially describe all particles and forces, including gravity, in a unified framework. Supersymmetry in this context is useful because it introduces a relationship between two major categories of particles: *bosons* and *fermions*. Bosons are force-carrying particles (like photons for electromagnetic force), while fermions make up matter (like electrons and quarks). Supersymmetry suggests that for every fermion, there is a corresponding boson, and vice versa. This relationship helps smooth out mathematical inconsistencies that would otherwise make string theory unworkable.

By including fermions in the framework with supersymmetry, string theory is able to describe both matter and forces. So in a sense, supersymmetry is the bridge that allows string theory to potentially explain all particles and forces in a single, coherent theory! (science supersymmetry, sd) (Introduction to Supersymmetry, 2010) (SUSY, sd) (physics and astronomy, supersymmery, 2006)

Vectors & spinors

Vectors are mathematical objects that represent both a magnitude (or size) and a direction. They are fundamental in physics and engineering for describing quantities that aren't fully defined by a single number alone. For example, velocity, force, and acceleration are all vector quantities because they have a specific direction in addition to their magnitude.

Spinors are a lot more complex, they describe particles with "spin," a fundamental quantum property that doesn't align with ordinary spatial rotation. Unlike vectors, spinors change in a unique way when rotated: if you rotate a spinor by 360 degrees, it doesn't return to its original state. Instead, it requires a full 720-degree rotation to come back to where it started. This strange property arises from quantum mechanics and is essential in describing particles like electrons, quarks, and neutrinos that possess half-integer spin values adding spinors makes particles gain full freedom.

In quantum mechanics, *spinors* provide a way to describe particles in terms that are consistent with both quantum mechanics and relativity. They are necessary for representing fermions, while vectors alone are typically used to describe bosons.

In string theory, spinors are introduced to describe the behavior of particles with spin in a higher-dimensional space, which is required for the theory to work properly and include all particles. They extend vectors to capture the subtler, multi-dimensional behavior of fermions, particularly under transformations like rotations. This extension is why spinors can eliminate certain issues and make it possible to include fermions in string theory, something that vectors alone cannot achieve. (A Child's Guide to Spinors, oxford, 2016) (zhelnorovich, 2016) (discussie van een document) (vectors, sd) (vectors and physics, sd)

Algebra

What are polynomials? polynomials are expressions in mathematics that consist of variables (like x or y) raised to various powers and combined using addition, subtraction, and multiplication. They're like mathematical "formulas" that can describe relationships, patterns, and shapes in different contexts.

A polynomial has "terms," where each term has a coefficient (a number) and a variable raised to an exponent. For example:

3x^2+2x+1

Here's how it breaks down:

3x^2: The first term has a coefficient of 3, the variable x, and an exponent of 2 (meaning x is squared).

2x: The second term has a coefficient of 2 and the variable x raised to the power of 1.

1: The third term is a constant, which means it doesn't have a variable. (:), sd)

Arithmetic operations are the fundamental operations of math like addition, subtraction, multiplication, and division. These operations allow us to combine numbers and variables.

The bulk

It reverences to a higher-dimensional space within our universe exists. Our 3-dimensional universe is in this larger-dimensional bulk, this bulk can have additional dimensions.

(Bron: Randall, L., & Sundrum, R. (1999). "A Large Mass Hierarchy from a Small Extra Dimension")

Quantum level

This level is about 1 nanometer (this is about 1 billionth of a meter) or smaller. This scale is used to study atoms and subatomic particles like electrons and protons.

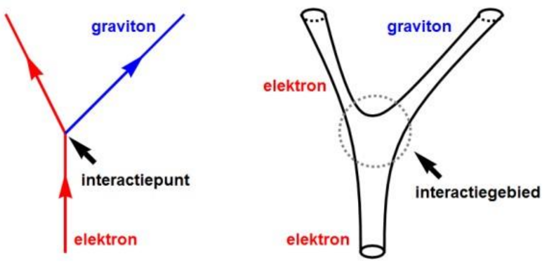
(Bron: Caltech's Quantum Science Explained.)

What is string theory?

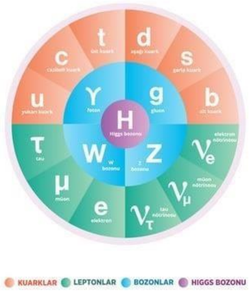
Let's start with explaining what the string theory really is. The string theory was made to get an understanding of everything, it's a unifying principle that we seek*: paradigm.* It has been made by multiple other stories bound together by ties and strings. Let us set an example, take a lyre as object for this, when you pluck one of its strings you get a set note when you pluck another one it will be a very different one. The mathematics of music is basically enough to explain the general idea of the string theory. But how do we go from Pythagoras his musical musings to formal proper physics?

The first thing that comes to the book is a particle accelerator, it is a machine that uses electromagnetic fields to propel charged particles to very high speeds. The tiny particles proceed to get smashed into each other producing high energy collisions , which are then studied closely. These experiments are the best way to study theories. Funnily these subatomic particles are nothing more than musical notes as Pythagoras predicted. If you could look very very closely at an electron you would see a rubber band, of course it's not an actual rubber band but something like a rubber band or rather said the strings in the string theory. And just like the strings on a lyre, when you put some energy into them they will vibrate. When you put energy in one way we call it a neutrinos, when you put energy in another way we call it an electron but after all it will be the same rubber band. When we look at dark matter we know it is what holds universes together, when looking at the universe from the view of the string theory we will see loads of rubber bands vibrating in different frequencies, but when the rubber band has higher octaves that we would call dark matter.

We now want to understand how these strings evolve through and around the universe we will use the same principles as the *standard model*. Let's say we throw a single electron at a target, the electron will propagate like a (sound) wave and as it reaches the target we will not know for sure where it will materialize. At the quantum scale the exact same experiment will give different examples, so we can only predict the probability of the chances where it will hit the target. To do that you will have to consider <u>all</u> possible scenarios and options. We sum all trajectories but also all possible interactions in the electron: an electron can emit a photon and absorb it later or, it can emit multiple photons and reabsorb it again. By summing al these possible scenarios we can obtain the desired outcome and the desired probability of that outcome. In the string theory it goes the same, however the particles in the model are no longer points but strings. The difference is great, a point traces a trajectory over time, but a string traces a surface leading to it having dimensions. So we will have to consider all possible geometries that the string can trace. As we told before different trajectories form different outcomes, like vibrating in different ways or duplicating itself and recombining itself, forming a geometry with a hole somewhere. So in this way the string theory not only predicts the existence of the graviton in the model but also calculates how it will interact with other particles. And so it describes quantum gravity.



Let’s continue on subject of the standard model. ’So our universe is made out of matter.. but what does that mean?’’ when we take a closer look at matter we will see that it is made out of particles there are different types of particles: electrons, quarks or neutrinos. Particles can interact with one another by exchanging other particles. All these different particles are accounted for, what we call the *standard model*. The standard model is a very precise mathematical description of the quantum world it contains.



It has two different categories of particles, one of which mostly constitutes matter: fermions and the other one which mostly describe the interactions: bosons

However for what we have explained before about the strings and the standard model now comes with a few problems. While we can predict bosons, there are a few problems which make predicting fermions nearly impossible. It mostly has to do with the rule of *supersymmetry (SUSY)*, String theory often relies on supersymmetry to naturally include fermions. Supersymmetry tells us that every boson has a corresponding fermion and vice versa. However, despite extensive searches, supersymmetry has not been observed experimentally. This discrepancy raises questions about the scale at which SUSY might manifest or whether it is a valid symmetry of nature at all.

One of the bigger problems now has to do with the dimensions and the model, our world has 4 dimensions, 3 dimensions of space and one of time. But the theory of the standard model predicts something very very different. The standard model that we have now can only exist in a world with 26 dimensions (the bosonic string theory). This is because it predicts the existence of a particle with imaginary mass called a "tachyon," which suggests an unstable, unphysical vacuum state that wouldn’t correspond to a stable, observable universe.

Another problem is: our world predicts another type of particle called : tachyon, this particle has an imaginary mass of $m = \sqrt{-1}$ and it’s obviously a mathematical problem that has to be removed.

To include fermions and get rid of tachyons we have to add something to our theory. That something is called *spinors*, we add these so called spinor on our strings so they gain freedom. Spinors are mathematical objects used to describe particles with spin, particularly fermions (particles like electrons, neutrinos, and quarks that make up the matter). Spinors are essential because they extend the concept of *vectors* to incorporate the intrinsic angular momentum of these particles in a way that's compatible with the principles of quantum mechanics and relativity.

So these spinners get rid of the tachyon and have it so we can predict fermions. But what do we do with the 3rd problem? Well by adding spinors the dimensions have shrunken to only 10, but that still hasn’t resolved our problem, and sadly with our theories right now we still can’t fully resolve it. By adding all these extra things we don’t call this theory the string theory anymore, but the M theory. (string lectures, sd) (SUSY, sd) (Ling-Fong) (Horejsi) (string theory, sd) (string lectures, sd) (string theory pdf) (standartmodel summary, 1998) (M Theory | Towards a theory of everything?, 2022)

What are dimensions?

So as we told you before the string theory (M theory) has 11 dimensions, and the string theory has 10. But what are dimensions? That’s a question that we are going to answer right here and right now.

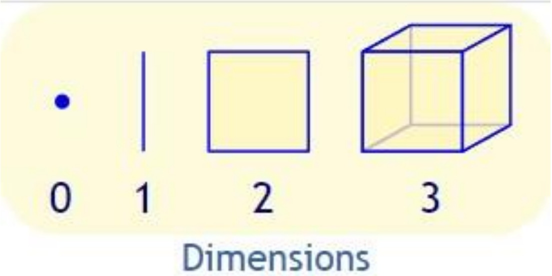
According to the physics fitting to our universe there are 3 dimensions of space and one of time, the first dimension described as length, the second one as hight and the 3rd one as depth. These dimensions are needed to explain how 3D objects work, however without the 4th one our universe can not exist as the 4th one is time. But why is it so that if the string theory is the most accurate we cannot see the extra 7 dimensions? If we include these 7 dimenions where are they? Well this is where compactified dimensions come into play. compactified dimensions are the extra spatial dimensions beyond the familiar three that we observe in daily life. Since we don’t observe these extra dimensions, the idea is that they are "compactified" or ‘’curled up’’ so small that they’re undetectable at human scales. To make it a bit easier to understand lets imagine an ant walking on a straw. The straw has 2 dimensions, the ant can walk back and forth and right to left around the circumference of the straw. Now if we zoom out enough, we won’t be able to see the depth and so it will look as if the ant can only walk back and forth in the length of the straw. The second dimension (left to right) is very small and so it cannot be seen from a large scale. In the string theory we can assume that the same thing occurs and that all of the rest of the dimensions are so small that they curl up on themselves.

This example also had the 4rth dimension: time. Without the 4th dimension we wouldn’t be able to perceive the framework within which events occur and objects exist.

So time is crucial because it forms a fundamental part of the fabric of spacetime. Spacetime is the concept that combines the three dimensions of space with time into a single, unified framework. Without the dimension of time, we couldn't describe the universe as we experience it, where things happen in a sequence: past, present, and future. Time governs the flow of events, meaning it helps us understand how things change and evolve. For instance, it allows us to see how a seed grows into a tree or how planets move through space. Without time, everything would appear frozen, and we wouldn't be able to perceive any change or motion.

Time also plays a vital role in our understanding of how the universe works. In the theory of relativity, time is linked with space to form spacetime, which explains how gravity works and how objects move. For example, the movement of planets around the sun or the bending of light around a black hole can only be explained by considering both space and time together. It explains how we (as in earth) circle around the sun alongside the other planets.

Dimensions can be explained in many ways, so we will start with the dimension of a point, dimension of a point is defined as the minimum number of coordinates that are needed to specify that point in it. We as being live in a 3 dimensional world, which is why we live in a world with 3 coordinates: with depth and length. These coordinates are needed to explain how our world looks in point:



People tend to think that we live in a 4 dimensional world because we are able to perceive a bit of the 4th dimension. However this isn't the case here, let's take a look at the 1th dimension, this dimension is like a line, if a 3 dimensional apple falls on this line they won't be able to perceive it as an apple but see it as a dot which gets bigger as the circumference of the apple gets bigger as it falls through this dimension. Because beings from a lower dimension won't be able to see beings from a higher dimension. This is also why we can ''see'' a little bit of time but not manipulate time itself as 4 dimensional beings would be able to do. Then you may ask: '' but are we able to manipulate the 3rd dimensions?'' the answer to this is, yes we are able to manipulate the dimensions. We are able to go left right up and down as we please, and have free play of the coordinates, if we consider the 4th dimension from the diametrical point of view then hypercube is a 4 dimensional object, as a cube has 6 square faces in the same way hypersurface of a tesseract. So the relationship between a normal cube and a tesseract is the same as a square and a normal cube.

So if 4 dimensional beings have control over time what do the 5 dimensional beings have control over? What are the other revealed dimensions if not time or one of the basic dimensions? well to explain this we have to move on from our 4 basic dimensions and dive into the spatical dimensions, the ones who were curled up on themselves without us being able to perceive any of them. The cause of this, of course they are small, so small that these additional spatial dimensions are hypothesized to be on the order of the Planck length, ~10−3510^{-35}10−35 meters. The 5th dimension, the first dimension that we aren't able to perceive even a little bit. We do have an idea of what it can be and how it works, it's the alternative realities like the ones in marvel and other hero movies. In these dimension, as the name suggests allows different possible worlds, where all the physical possibilities are realized. In some interpretations, it represents different quantum states. So the 5th dimension has to do with different realities, it is associated with different realities and alternate timelines. Just as the 4th dimension can use time to bend to their will, the 5th dimension allows beings to manipulate time to a much greater extent. Beings in the 5th dimension have the ability to create new timelines, instead of being confined to a single one. They can coexist in two timelines simultaneously, giving them a unique form of existence across multiple realities. Let's set an example, if a mistake was made in the past and they wish to alter it, 5 dimensional beings could change their decision and continue along a different timeline. Their lives would then adjust according to the different versions of themselves in these alternate universes. However, a key limitation for these beings is that they cannot directly interact with or meet their alternate selves from other universes.

The concept of the 5t dimension is supported by the idea that this universe unifies all fundamental forces of nature: electromagnetism, gravity, and the strong and weak nuclear forces. Too explain how these forces are integrated, the introduction of a 5th dimension was necessary. This is where the Kaluza-Klein theory comes into play. Oskar Klein proposed that light could be understood as a disturbance, like ripples in a higher-dimensional space. So as an example, imagine a fish swimming in a pond. When it rains, ripples form on the water's surface due to the raindrops. However, the fish can only perceive these ripples as shadows on the surface, without fully grasping the cause. In a similar way, gravity and electromagnetic forces, which seem unrelated in our 4 dimensional perspective, can be seen as interconnected when viewed from the higher dimensions proposed by string theory and M-theory. Superstring theory further supports this idea, suggesting that these forces are indeed related at a deeper, multi-dimensional level.

While the 5th dimension allows for movement between universes with the same laws of physics but different timelines, the 6th dimension provides access to all possible universes with different starting conditions and fundamental laws. So parallel universes exist in the 6th dimension but different from the 5th dimension, the 6 dimensional beings can see everything that's happening in the made alternative universes. So it can also see the futures and pasts of those same universes. it does have its limitations, all parallel universes must have the same beginning. That is for instance the big bang. One of string theory's main goals is to unify all fundamental forces within a single framework. The early universe, as described by the Big Bang, is the context where such unification is most likely to occur due to the extremely high temperatures and energies present shortly after the Big Bang. That's why without the big bang the concept of the unifying forces is so important and why the 6th dimension wouldn't exist without the same beginning.

The 7th dimension is where someone can access different universes with entirely different initial conditions so unlike the 6th dimension it doesn't need to have the exact same start. This means that it's not just about having different outcomes (like in the 5th and 6th dimensions), but starting with completely different physical settings and fundamental laws of nature. These worlds don't have to rely on the rules set by fundamental laws of nature like gravity. So, if the sixth dimension allows for observing all the possible outcomes of a universe that began with the same physical laws and constants (such as gravity and electromagnetism), the seventh dimension opens the door to universes that begin with different sets of laws. These universes may have started with whole different big bang conditions or may operate with different physics, such as alternate values for the speed of light, or forces that behave differently from our own. The 7th dimension is a very important one because it's the dimension that allows us to explore how different universes could function based on different starting points. If our universe's laws of physics are fine-tuned for life, the seventh dimension would let us see how dramatically different other universes could be. This is crucial for understanding ideas like the *multiverse*, where countless universes could exist, each with its own unique set of rules. However when by example the speed of light is altered the beings in the 7th dimension can only access other alternative worlds with these same altered physics. So in the 7th dimension, you cannot access universes where the basic laws of nature (such as gravity, electromagnetism, or quantum mechanics) are fundamentally different. This is a significant limitation, as the seventh dimension is limited to exploring universes that are variations of our own in terms of starting conditions, but not in the rules that govern those universes.

In the Eighth Dimension, you could access a universe where not only are the physical constants different, but where the forces of nature themselves may be altered or even nonexistent. For example, in one universe, electromagnetism could behave in completely unfamiliar ways, or gravity might repel rather than attract. So imagine a universe that has more than three spatial dimensions and where life forms are higher-dimensional beings. Alternatively, there could be universes without time or where entirely new forces exist, outside of the four known forces in our universe (gravity, electromagnetism, the weak nuclear force and the strong nuclear force).

Another example is a universe where the laws of quantum mechanics don't apply, or where there's a completely different way of constructing matter that doesn't rely on atoms or subatomic particles as we know them. It goes beyond the 7th dimension, allowing for universes that operate with entirely different principles of reality, where everything from space and time to the forces of nature could behave in ways that are fundamentally different from what we know. It’s essential for theorizing about the most extreme possibilities of the multiverse and understanding how vast the scope of potential realities could be!

In the ninth dimension, you can explore all possible universes with different initial conditions, physical laws, and most importantly, different dimensional structures. This means that in addition to universes with different laws of physics (as allowed in the eighth dimension), the ninth dimension introduces the possibility of universes with more or fewer spatial dimensions or even different temporal dimensions.

In simpler terms, the ninth dimension allows for universes where the very nature of space and time itself could be fundamentally different. While the eighth dimension lets you explore universes with different physical laws but the same three spatial dimensions and one time dimension, the ninth dimension expands that by allowing you to move through universes where the number of spatial and temporal dimensions can vary. However, all the universes must be mathematically consistent and logically coherent. This means that, although the laws of physics and dimensions may be different between universes, those laws must still allow for stable, non-contradictory universes. So you cannot enter a universe where the 1 + 1 = 3 is really true.

So when do we gain absolute freedom? That is, within the 10th dimension, the first dimension of absolute chaos. The 10th dimension is the most abstract and complex concept in string theory and M-Theory, representing the ultimate level of the multiverse. At this stage, all possible realities, dimensions, and universes are encompassed. The tenth dimension contains everything, it is the highest possible dimension, beyond which no further dimensions exist, as there is nothing left to include. Apart from the 11th dimension but that’s something we explain in the other sub question.

The 10Th dimension has absolutely zero restrictions. The 10th dimension includes realities and universes so far removed from human understanding that they may be impossible to conceptualize or visualize. For example, universes where there is no time at all, or where space is a completely different entity than we understand, would be part of this dimension. (10 dimensions and more, sd) (Curling Up Extra Dimensions in String Theory, sd) (why string theory requiers extra dimensions, sd) (why does string theory require 9 dimensions of space and one dimension of time?, 2016)

Where are these additional dimensions?

There are multiple theories about where the dimensions are hidden. We live in a 3-dimensional world, but there are more than 3 dimensions, so where are they? We will be explaining 3 theories of where these dimensions are. One of the many theories is also explained in the sub-question; 'What are dimensions?’, the theory that has been explained there is the theory of compactification, so we won’t go into that theory in this sub-question. The 3 theories that we will be explaining are large extra dimensions, multiverse/many worlds hypothesis and the holographic principle.

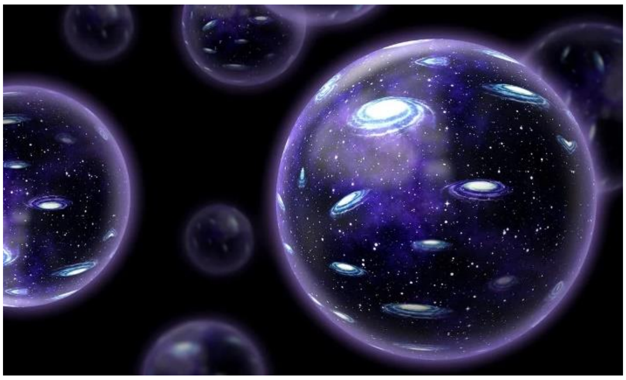
Large extra dimensions

We will start by explaining large extra dimensions. This theory goes in on why gravity is such a weak force. Gravity is the weakest force out of the 4 fundamental forces we know (the weak force, the strong force, the electromagnetic force and gravity). It's so weak that when you have a paperclip and a small magnet, the paperclip will get stuck to the small magnet instead of falling down. This means that even though gravity is pulling on it with the mass of the entire Earth, the small magnet is still holding on to the paperclip. This is weird, because gravity should be stronger as it’s one of the fundamental forces. This weakness of gravity compared to the other forces is also known as the hierarchy problem. This theory explains why it’s so weak. It can spread out into additional dimensions. This theory proposes that the dimensions aren’t as tiny as we thought, they can be up to 1 millimeter big (this might not seem big, but this is really big on a particle scale). As I wrote earlier, the gravity force spreads out, this would be an explanation to why the force is so weak. It works as follows, imagine we live on a 2-dimentional piece of paper, floating in a 3-dimentional room. So, if we were to live in 2 dimensions, we would be able to only go left, right, front and back, we wouldn't realize the third dimension (going up and down). According to this theory, forces like electromagnetism, the strong force and the weak force are confined to this 2-dimensional brane, kind of like how ink is stuck on the paper. These forces stay in the 2-dimensional space and don’t interact with the surrounding 3-dimensional room. Gravity is an exception here, gravity isn’t trapped on the piece of paper, it can leak into the 3-dimensional space. Now imagine it with our 3-dimentional world and instead of a piece of paper, it is called a brane (we live on a 3D brane), and instead of the room, we call it the bulk. So, because gravity can go in these extra dimensions, we experience gravity weaker on our brane. Because of this gravity might be a strong force, but it seems weaker on our brane, because it spreads evenly across these extra dimensions in the bulk.

(Bron: Arkani-Hamed, N., Dimopoulos, S., & Dvali, G. (1998). *The Hierarchy Problem and New Dimensions at a Millimeter*. **Physics Letters B, 429(3-4), 263–272.**)

multiverse

The second theory is multiverse/many worlds hypothesis. This theory goes in on the multiverse, so what is that? We live in our universe, but according to this theory there are more universes. There could be a lot of universes, this is called multiverse, but also parallel universes or alternate realities. These universes could have different versions of ourselves, different dimensions, different physical laws, it could be the opposite universe of what we have right now. As you know, for string theory to make sense, it requires to have extra dimensions. These dimensions are curled up and incredibly tiny. The different kinds of arrangements of these extra dimensions are countless, each arrangement can affect the behavior of particles, the forces, their strength, even the laws of physics, like the gravitational force and even the speed of light. Each of these different arrangements leads to a different universe with its own unique laws of physics, gravitational force and speed of light. This could also mean that a different universe could have a different number of spatial dimensions. We are very lucky that life itself is possible in our universe, there could be countless universes where life is not possible.



(Bron: The Fabric of the Cosmos: Universe or Multiverse HD & HQ (YouTube))

holographic principle

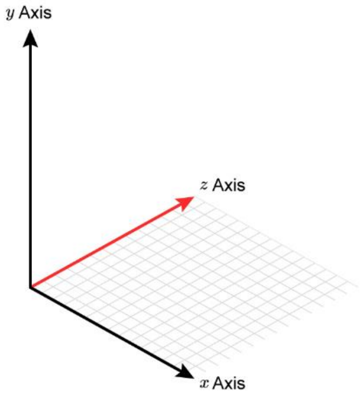
The last theory that we will be explaining is the holographic principle. This theory suggests that our 3-dimensional universe may be a ‘projection’ of information stored on a 2-dimensional surface, kind of similar to a hologram. This 2-dimensional surface might exist at the edge of the universe or near certain areas, like black holes. This theory was discovered because of black hole physics. Scientists discovered, the maximum amount of information that a black hole can hold does not depend on the volume of a black hole, but on its surface area. With this surface area is meant the event horizon, this marks the point of no return, past this surface and nothing, even light, can’t go back. This discovery led to the idea that all the information on what's inside a black hole can be stored on its 2-dimensional surface. The theory developed from here, it proposes that the whole universe might work similarly. Everything we see in a 3-dimensional space could come from information stored on a 2-dimensional surface. This could mean that our 3-dimensional world is just like a hologram projected from 2-dimensional data. So, where are these extra dimensions? They may exist on the edge of the universe, where the 2-dimensional surface may also exist, it stores all the information that is creating our 3-dimensional experience. Although we can't directly perceive these extra dimensions, they still affect the laws of physics, particularly in extreme conditions like those near black holes. Or they could be curled up so small that we can't interact with them.

(bron: PBS Space Time’s episodes: The Holographic Universe Explained and Does Space Emerge from a Holographic Boundary?)

In what way does the z-axis help to illustrate the concept of additional dimensions?

To understand the z-axis and t-axis, I first need to explain what the x-axis and y-axis are. An axis is basically an imaginary line, it is used to find a position and direction in space. To start with the x-axis,

you might be familiar with it. The x-axis is a horizontal line that represents the first dimension. Each point tells us how far to the left or right we are from the center. When you go from a 1-dimensional space to a 2-dimensional space you get the x-axis and y-axis. The x-axis is the same and the y-axis is vertical, this means it goes up and down. So, each point tells us how far up and down we are from the center. If we have the point (4, 3), the number 4 is for the x-axis and tells us its 4 points to the right form the center, the number 3 is for the y-axis it tells us its 3 points up from the center. When we add another dimension and go to a 3-dimensional space, there will be a new axis, the z-axis. This axis adds dept to a 3-dimensional object, therefore the z-axis goes forwards and backwards, this is seen in the picture. So far, we have the x-axis; this changes position left or right. The y-axis: these changes position up or down, and we have the z-axis; this changes position in dept, this moves the object closer or further away from the viewing point. When we go to a 4-dimensional space, there is a new axis, the t-axis. This goes for every higher dimension, every time you add a dimension, you add an axis as well. Each axis represents a direction in which we can navigate through or measure a space.



What are the mathematics and basic physics of string theory and its dimensions?

We have been talking a lot about these dimension, but what lies inside those dimensions or what really are they? Dimensions and Coordinate Systems are the starting point. A dimension in mathematics is essentially an independent direction along which a point or object can be placed. In everyday life, we're familiar with three dimensions, length, height, and depth, The 4th dimension is time, which lets us sequence events and perceive change. For instance, imagine a two-dimensional surface like a sheet of paper. Higher dimensions can exist but may be unobservable if they're outside our direct experience. For each dimension, we add another coordinate in mathematical space e.g, x, y, z, t for 3D + time. Higher dimensions in string theory add even more coordinates, like x_1, x_2, x_3, ..., x_{10} to represent directions in a ten-dimensional space.

Ricci curvature

So a dimension is a fundamental concept in mathematics used to describe the position of a point in space or the extent of an object along a specific axis. Dimensions provide a way to quantify the size, shape, and position of objects in various spaces such as lines, planes, and volumes. To help us explain dimensions a bit we will first use the Ricci curvature and flat to help! We will explain it in simple matters that won't need us to go into detail but do explain the big part. So what is Ricci curvature? To say it simply it is a measure of how much space is curved in a specific way that affects volumes. Let's imagine a small ball in space, Ricci curvature tells you how the volume of that ball would differ if space were flat versus if it were curved.

If that's not clear imagine the table is perfectly flat, like a smooth piece of paper. You put a small pen circle on it. The circle's area is exactly what you'd expect based on its size because the surface doesn't change it at all. This is the uncurved obviously, so let's try the one where there is curvature. If you draw the same circle with your pen, but this time on the curved surface of a ball, the space inside the circle changes. If the ball is squished in some way or has a bump, the space inside your circle isn't quite what you expect anymore, sometimes it feels smaller, sometimes bigger, depending on how the ball curves. Now let's continue, so Ricci curvature is a measure of how much space is curved in a specific way that affects volumes. Ricci curvature tells you how the volume would differ if space were flat versus if it were curved.

Mathematically, the Ricci curvature tensor Rmn is derived from the more detailed Riemann curvature tensor, which fully describes how space curves in all directions. The Ricci curvature is used in einstein's theory of general relativity, which it in the Einstein field equations appears : $Rmn = -\frac{1}{2}Rgmn = 8\pi\,\mathrm{GTnm}$

R_{mn} is the Ricci curvature tensor.

R is the Ricci scalar, a single number summarizing the overall curvature. g_{mn}

is the *metric tensor*, describing the shape of space.

T_{mn} is the *energy-momentum tensor*, which tells us how matter and energy are

distributed.

As we told you before in string theory, there are more than just the 3 spatial dimensions we experience; there are extra dimensions that are compactified into tiny shapes, often using special geometries called *Calabi-Yau manifolds*. A key requirement for these compactified spaces is that they be Ricci-flat: $R_{mn} = 0$. This means that the Ricci curvature is zero, indicating no "net" bending in these extra dimensions. Without this the curled up dimensions wouldn’t exist.

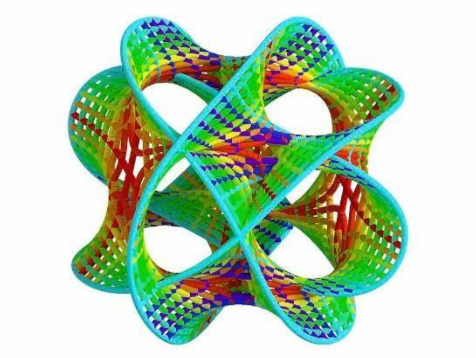
Ricci Curvature helps quantify this concept by measuring how space bends around a point, which affects how volumes behave within that space. In higher dimensions, Ricci curvature quantifies this bending of space. The Ricci curvature tensor measures how a volume near a point differs in curved space compared to flat space. In string theory, Ricci-flatness where Rmn= 0 is a requirement for compactifying extra dimensions without impacting our observable 3D world.

Ricci-flatness also provides stability for extra dimensions. Ricci-flat spaces are special curved spaces with no net curvature in any direction, allowing extra dimensions to curl up without affecting large-scale reality. Let’s say a ball was rolling perfectly on a curved, frictionless path. It can keep rolling unaffected by any net forces due to the path’s balanced curvature. In string theory, the Ricci-flat property of extra dimensions keeps them stable in a compactified state. A Ricci-flat condition, represented by Rmn=0 balances curvature in the extra dimensions, stabilizing them without causing observable changes in our 3D space. This requirement is similar to properties seen in general relativity, where Ricci-flat conditions help define spacetime shapes in the absence of matter.

This stability relies on a core mathematical structure, *the Metric Tensor* g_{mn}, which describes distances and the shape of space in any dimension. The metric tensor functions like a set of rules that define how to measure length, angles, and volume. In the higher dimensions, the metric tensor adapts to include additional coordinates, helping define shapes and distances for spaces beyond three dimensions.

Calibi yau manifolds

We also have mentioned calibi yau manifolds a lot, but what is it? Well it has to do with the left over curled up dimensions.t he compactified dimensions are often structured as *CalabiYau manifolds*. These are complex shapes that are particularly well-suited for string compactification because they have a high degree of symmetry and mathematical stability, which help maintain *supersymmetry* (a symmetry between particles and forces) when the theory is reduced to four dimensions.



Calabi-Yau manifolds are particularly useful our in string theory because they allow for a wide range of vibrational modes for strings, each corresponding to different particle types. The exact shape of the Calabi-Yau manifold used for compactification has a direct impact on the resulting physical theory. Each possible shape can be thought of as a different configuration of compactified dimensions, and this variety of shapes leads to different possible low energy theories, each with its own set of particles and forces. Thus, selecting a particular Calabi-Yau manifold determines the specific properties of the particles, forces, and interactions that we observe in our four-dimensional universe.

Einstein's Field Equations

further explore the structure of space, linking spacetime geometry to energy and matter. These equations form the foundation of general relativity and gravity's behavior, both in observable and higher-dimensional space. The analogy explains gravitational attraction. similar curvature may arise from extra-dimensional interactions. The field equations involve the *Ricci curvature tensor*, the *Ricci scalar (*R), and the energy-momentum tensor Tmn describing how space bends in response to matter.

Kaluza-klein theory

The goal is that of unifying all forces, *Kaluza-Klein Theory* offers a bit more insight to how an extra dimension could merge different fundamental forces. in Einstein's general theory of relativity, gravity is depicted as the curvature of four-dimensional spacetime. Kaluza took this idea further by adding an extra, fifth dimension to this model, creating a 5D space where particles and fields would interact not just within our usual 4D framework, but also in this tiny fifth dimension. However, like we told you before this additional dimension is so small, on the scale of the Planck length (10−3510^{-35}10−35 meters)that it is compactified or curled up into a minuscule loop. Due to its size, it's invisible to us, which explains why we don't observe it directly.

When Kaluza introduced this fifth dimension, he noticed something remarkable: the resulting equations describing the geometry of this 5D spacetime accounted for both gravity and electromagnetism. The curvature equations not only included terms for the usual gravitational interactions but also contained terms that exactly corresponded to *electromagnetic fields*. Essentially, by adding a fifth dimension to spacetime, Kaluza's model naturally produced a unified description of these two forces, encapsulating both gravity and electromagnetism into a single framework.

a fifth dimension enables the unification of electromagnetism with gravity, suggesting that unseen dimensions might combine forces that appear distinct in our 4D world. For instance, think of a fish in a pond that only sees surface ripples, unaware of the raindrops causing them. To the fish, ripples (or forces) seem unrelated, but in higher dimensions, they could be linked. Kaluza-Klein theory combines the metric tensor for 4D space with an additional dimension, establishing a foundation for understanding how forces might be unified through extra dimensions.

Algebra and group theory

Lastly, *Algebra and Group Theory* play a role in describing symmetries in higher dimensions. In string theory, group theory captures the symmetries and transformations of vibrating strings which we explained before, with each vibrational mode corresponding to a particle type. Each mode, like different notes on a lyre, depends on the string's tension, length, and shape. For strings, these modes determine particles' properties, and group theory formalizes these "notes" to ensure consistency across dimensions. Symmetry groups like SU(3), SU(2), and U(1) describe particle interactions and ensure the mathematical structure of string theory remains valid in multiple dimensions.

So algebra and group theory are powerful branches of mathematics that allow us to describe and manipulate abstract structures and symmetries, which play important roles in our understanding of higher-dimensional physics. In string theory, where dimensions beyond our familiar three-dimensional space are proposed, algebra and group theory provide the ''language'' and tools needed to represent these complex, multi-dimensional relationships, symmetries, and transformations. Each branch of mathematics, while distinct supports the other, creating a very strong framework for exploring the nature of reality beyond observable dimensions.

Algebra is about using symbols (like x and y) to represent quantities and relationships between them. Let's start simple with an equation like $3x+4=10$ it makes easy to understand how we get to the answer. However algebra can handle far more complex relationships, making it perfect for physics. At its core, algebra uses variables to create equations, which can model everything from simple motion to the behavior of complex systems. For example, in physics, an equation might describe the relationship between force, mass, and acceleration:

$$F=Ma$$

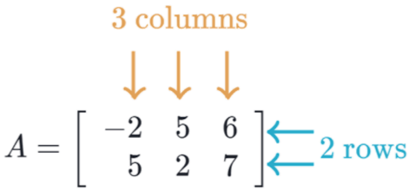
In this formula, F (force) depends on m (mass) and a (acceleration). Algebra lets us rearrange this formula to solve for any one of these values based on the others. So if we know mass and acceleration, we can figure out force.

In higher-dimensional theories like our string theory, variables become even more powerful. Instead of just dealing with 3D space, we may have to work in 10 dimensions. Algebra gives us a way to represent these dimensions and the relationships within them. Imagine trying to describe the paths of moving objects in our 3D world, a relatively straightforward task using algebra. Now imagine trying to do the same thing in 10dimensional space. Algebra gives us tools like *matrices*, which are grids of numbers representing multiple values and dimensions. A *matrix* can represent a transformation that would be very hard to visualize, such as a rotation in five-dimensional space.

An algebraic expression, like 3x2+2x−5, combines variables and constants using *arithmetic* operations, but it doesn't necessarily set an equality. An equation, on the other hand, introduces an equality, such as 3x+5=2x−1, creating a relationship that can be solved to find the values of x that satisfy it. Algebraic structures, such as *polynomials and fields*, (these are explained in the theoretic statics)but also matrices, provide powerful ways to represent complex ideas.

matrices

Well explain a bit more about *matrices* before we continue with the further explanation algebra itself. A matrix is an organized grid of numbers arranged into rows and columns, forming a rectangular or square shape. Each entry, known as an *element*, can be any type of number, including positive or negative integers, fractions, decimals, or even complex numbers. This flexibility allows matrices to represent a vast range of mathematical, scientific, and engineering problems.

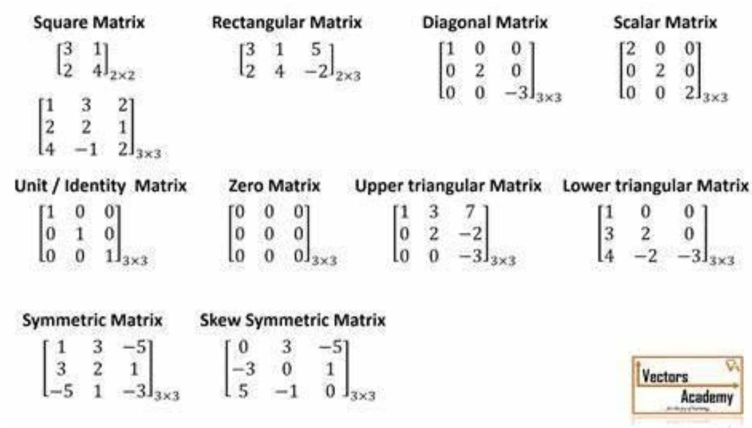


In algebra, matrices are typically represented by bold or underlined uppercase letters (**A** or *B for example*) and the collection of elements is often enclosed within square brackets, like , or sometimes curved parentheses, but not curly braces. A matrix is identified by its dimensions, defined by the count of rows (horizontal lines) and columns (vertical lines). For example, a 2x3 matrix has 2 rows and 3 columns (as you can see in the picture above).

aij represents the element located in the i-th row and j-th column of a matrix A. This notation is a way of indexing elements in a matrix so that each entry has a specific position, making it easy to refer to any individual element in a structured way:

$$\begin{array}{lccc} & 11a & 12a & 13a \\ \text{A} = & 21a & 22a & 23a \\ & 31a & 32a & 33a \end{array}$$

We have a lot of different types of matrices here are a few of the most basic examples:



- *square matrix*
- *rectangular matrix*
- *diagonal matrix*
- *scalar matrix*
- *identity matrix*
- *zero matrix*
- *upper triangular matrix*
- *lower triangular matrix -*
  Symmetric matrix
- *Skew symmetric matrix*

A square matrix : is one where the number of rows is equal to the number of columns, like a 2x2 or 3x3 matrix.

A *rectangular matrix* has different numbers of rows and columns, such as a 2x3 matrix or a 3x2 matrix.

In a *diagonal matrix*, all elements off the main diagonal are zero. As you can see in the 3x3 example in picture. Only the main diagonal contains values, and the rest are zero.

A *scalar matrix* is a type of diagonal matrix where all diagonal values are the same.

A *identity matrix* is a square matrix where all elements on the main diagonal are 1, and all off-diagonal elements are 0.

A *zero matrix* speaks for itself so it is one in which all elements are zero.

In an *upper triangular matrix*, all elements below the main diagonal are zero.

A *lower triangular matrix* is the opposite of an upper triangular matrix, all elements above the main diagonal are zero.

A *symmetric matrix* is a square matrix that mirrors itself across the main diagonal. This means that aij=aji for all entries.

A *skew-symmetric matrix* is a square matrix where aij=−aji and all diagonal elements are zero.

In our 3 dimensional space, we only need three coordinates to specify a position. However, in string theory space is considered to have up to 10 (or even 11) dimensions. When working with such high-dimensional spaces, it becomes extremely challenging to manage all the relationships, transformations, and properties within each dimension. Matrices allow physicists to represent these higher-dimensional spaces in an organized, manageable form.

For instance, if we consider a 10x10 matrix, each row and column in this matrix can represent one of the dimensions in string theory. Each entry within this matrix can describe specific relationships or transformations between those dimensions. This helps us build a structured map of how these dimensions interact, which is key for exploring the theoretical properties of space-time in string theory. If we look at the standard model, it was set up by using matrices, this was done in such a way that each entry in a matrix could represent a unique aspect of a string's vibrational mode, such as its frequency or amplitude. When we use matrices to organize these details, they help structure the relationships between particles, encode probabilities for particle transformations, and establish mass and interaction patterns among particles. Through *gauge theory*, mixing matrices, and the *Higgs mechanism*, matrices make it possible to describe the symmetries, interactions, and decay processes central to particle physics. This makes matrices essential for exploring how various strings' vibrations lead to the diversity of particles observed in nature.

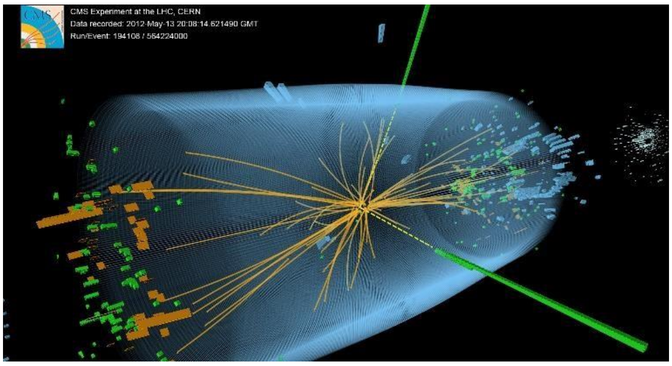
Fields

Okey enough about matrices lets continue with the rest. But what more is there to explain? Weve told you a lot already. Remember how string theory aims to unify the four fundamental forces in nature within a single framework? Each of these forces correspond to a *field*, and string theory treats these forces as arising from the interaction of strings in multiple dimensions. Fields thus become the medium for expressing how strings interact and influence each other across different forces. In physics, *a field* is a region in which each point has a physical quantity associated with it. This quantity could be a simple number, as in the case of *scalar fields* like *the Higgs field*, or a directional *vector,* as in fields associated with forces, such as the gravitational or electric fields. Fields serve as the fundamental basis for understanding interactions in the universe, explaining how forces act over distances and how objects interact even without direct contact.

For example, *gravitational fields* explain why objects fall to the ground. Earth's gravitational field exerts a force on objects, pulling them toward the center of the planet, and this force is what we perceive as gravity. Similarly, a *magnetic field* is the force-producing region around a magnet. If a paper clip is placed in a magnet's magnetic field, it will experience a pull toward the magnet, and two magnets with like poles will repel each other when placed within each other's magnetic fields. *Electric fields* function similarly: an electric charge generates a field around itself, and any other charged particle placed within this field experiences an attractive or repulsive electric force. The strength of these forces in any given region is often represented by *field lines*; the closer the lines, the stronger the force within that part of the field. Field lines help visualize the nature of the force, showing both direction and strength across space.

We have mentioned a lot of types of fields, especially *Higgs* keeps coming back on word, so let us explain that first. The *Higgs field* is a fundamental component of the Standard Model of particle physics, providing the mechanism by which particles get a mass. Let's set an example, imagine the Higgs field as an invisible, omnipresent field that fills the entire universe. Like other fields, each point in space has a physical quantity associated with it, giving every part of the field a consistent value. This is what makes it a *scalar field*, meaning it has magnitude but no direction. Unlike *vector fields*, such as gravitational or magnetic fields, where quantities like force act in a particular direction. The Higgs field affects particles uniformly by interacting with them as they move through it, and defining how much mass they will have.

The concept of mass generation through the Higgs field is essential to the way particles behave and interact. Different particles interact with the Higgs field in varying degrees. Some particles interact strongly, such as the W and Z bosons, which play a role in the weak nuclear force. These particles acquire a significant amount of mass from the field. However particles like electrons interact more weakly with the field, so they have relatively small masses. Certain particles, such as photons do not interact with the Higgs field at all, which s why they remain massless. To get a better view of how this works imagine particles moving through the Higgs field as if they're traveling through a thick fluid. Some particles feel more resistance as they pass through the field, slowing them down and giving them a quality we observe as mass. Particles that experience more resistance "feel" the field more strongly, thereby acquiring greater mass. This concept is often illustrated by comparing movement through the Higgs field to the experience of wading through water: the more resistance, the slower the particle, and the more mass it appears to have. This "resistance" effect is intrinsic to how mass arises in our universe.



The Higgs boson is the particle associated with the Higgs field, acting as the smallest quantum unit or "particle" of this field in the way that photons are the particles of the electromagnetic field. The detection of the Higgs boson provided direct evidence for the Higgs field's existence, confirming the long-held theory that the Higgs field permeates the universe and interacts with particles to grant them mass. Without the Higgs field, particles would be massless, atoms couldn't form, and matter would not exist in the way it does today. Basic forces like gravity would function differently, profoundly altering the fabric of reality. The Higgs field, therefore, plays a foundational role in determining the properties of matter. (the Higgs boson, 2023)

So in string theory, fields such as the Higgs field are viewed in terms of the vibrational states of strings. The Higgs field represents one specific vibrational mode, providing insights into how fields contribute to particle characteristics like mass. In this way, the Higgs field serves as a bridge between the physics of the Standard Model and the deeper questions string theory attempts to answer about the nature of the universe and the forces governing it. This connection highlights why the Higgs field is not only a critical element of particle physics but also a doorway to exploring new dimensions and fields that may redefine our understanding of the universe.

Now what are the other types of fields? A *scalar field* is the simplest type, associating a single value with every point in space. The value doesn't have direction but only magnitude. Scalar fields are common in physics, examples include *temperature fields* and *potential fields*. In particle physics, the Higgs field is a scalar field that interacts with certain particles to give them mass. When particles move through this field, they gain a property we interpret as mass, which is essential in the Standard Model of particle physics.

Vector fields are a bit more complex, assigning both a magnitude and a direction to each point in space. Gravitational and magnetic fields are examples, with direction and strength varying from one point to another. In a gravitational field, the vector points toward the source of gravity, like Earth, and represents the force acting on objects due to gravity. In an electric field, vectors indicate the direction in which a positive charge would move if placed within the field. The combination of direction and intensity makes vector fields essential for understanding how forces operate at a distance.

Tensor Fields extend the concept of scalars and vectors to even more complex properties, allowing us to describe forces and structures in multi-dimensional ways. Tensor fields are crucial in general relativity, where Einstein's equations describe gravity not as a force, but as a curvature in spacetime. This curvature is represented by a metric tensor field that describes how space and time are stretched or compressed around massive objects, like planets or stars. The tensor field approach explains gravitational interactions on a cosmic scale and is key to understanding black holes, gravitational waves, and the expansion of the universe.

Quantum field theory takes the field concept into the quantum realm, describing particles as excitations or ripples in underlying quantum fields. Each type of particle, such as electrons or quarks, corresponds to a specific quantum field. For instance, the electron field and photon field describe how electrons and photons exist and interact. Quantum fields explain fundamental forces through particle exchange; for example, photons are the force carriers of the electromagnetic field, enabling charged particles to interact at a distance.

Group theory

We have finished fields, and have ended the stuff about algebra, now we go into group theory. In string theory, group theory is not simply a tool for understanding symmetry, it is the mathematical framework that fundamentally organizes how strings interact, how physical fields arise, and how compactified dimensions shape observable physics. To understand string theory’s reliance on group theory, we need to look beyond common notions of "curled-up" dimensions and enter the language of *gauge groups, gauge symmetries, modular invariance, representation theory, compactification (what we have already told you about), dualities, and D-brane dynamics*. Each of these concepts relies heavily on group-theoretic structures, and they weave together to form the complex tapestry of string theory’s predictions.

Gauge groups.

So gauge groups are a fundamental concept in group theory, which is used to describe the symmetries of physical systems and the interactions of fundamental forces. A gauge group is a mathematical group that defines how fields in a theory can transform under local gauge transformations. These transformations can vary from point to point in space and time, reflecting the local nature of the symmetries involved.

To understand *gauge groups*, we start with the concept of a *group* in mathematics. A group is a set of elements along with an operation that combines any two elements to form a third element. This operation must satisfy four key properties: closure, the result of the operation is also in the group. Associativity, the way in which elements are grouped does not affect the result. The presence of an identity element, an element that does not change other elements when combined, and the existence of inverses, for every element, there is another that combines with it to yield the identity element. In the context of physics, gauge groups represent the symmetries of the laws governing physical systems. Many gauge groups are *Lie groups*, which means they are also smooth manifolds. This property allows us to perform calculus on these groups, making them useful for continuous transformations. For example, the group *SU(2)*, which describes symmetries involving rotations in a 2 dimensional complex space, can be represented by matrices that maintain certain properties. The generators of a Lie group are a set of basic elements of the corresponding Lie algebra and describe infinitesimal transformations. For example, for the *SU(2)* group, the generators can be represented by the *Pauli matrices*.

Gauge groups can be classified into *local* and *global symmetries*. A global symmetry applies uniformly across space and time, meaning the same transformation is applied everywhere. In contrast, a local symmetry allows the transformation to vary from point to point. Gauge theories are based on local symmetries, which require the introduction of gauge fields to ensure that physical predictions remain unchanged under local transformations.

One of the simplest examples of a gauge group is *U(1),* which is associated with electromagnetism. In this case, the gauge symmetry involves transformations of the form:

$$\psi(x) \rightarrow e^{i\theta(x)}\, \psi(x)$$

Here, $\theta(x)$ is a function that can vary with position x, and $\psi(x)$ is the field representing charged particles. The gauge field associated with this transformation is the electromagnetic field $A\mu$.

Another important gauge group is *SU(2),* which describes the weak nuclear force. This group involves transformations that can mix two components of a field. The weak interactions are mediated by the W and Z bosons, which arise from this gauge group. The generators of SU(2) can be represented by three matrices, which correspond to the three types of transformations in this group.

The *strong nuclear force* is described by the gauge group *SU(3).* This group has eight generators, corresponding to the eight types of gluons that mediate interactions between quarks. The gauge fields in this case are the gluon fields, responsible for holding quarks together to form protons and neutrons.

To ensure that the physics remains unchanged under these local gauge transformations, we introduce the covariant *derivative* $D\mu$,

$$D\mu=\partial\mu+igA\mu(x)$$

In this expression, $\partial\mu$ is the usual derivative, $A\mu$ is the gauge field associated with the gauge group, and g is the *coupling constant ,* The coupling constant is a numerical factor that appears in the interaction terms of a quantum field theory. It governs the strength of the interaction between different fields or particles. The covariant derivative allows us to maintain gauge invariance, ensuring that the physical laws derived from the theory do not depend on the specific choice of gauge.

Gauge symmetry.

Gauge symmetry refers to a type of symmetry where the equations describing a physical system remain unchanged (invariant) under specific transformations of the fields. These transformations can vary from point to point in space and time, known as *local transformations*. Unlike global symmetries, which apply uniformly across all space, gauge symmetries allow different parts of the system to transform independently. the gauge group is SO(32). This symmetry dictates the behavior of particles and the types of interactions that can occur.

If we consider a field $\psi(x)$ that transforms under a gauge transformation as follows:

$$\psi(x)\rightarrow\psi'(x)=U(x)\psi(x)$$

Where, $U(x)$ is an element of the gauge group that can vary with position x. For example, in U(1) gauge theory, $U(x)=e^{i\theta(x)}$, where $\theta(x)$ is a real-valued function.

Worksheet dynamics.

Picture a guitar string being plucked. As the string vibrates, it moves up and down and side to side. Each position of the string at a given moment in time represents a point on the worldsheet. If we were to draw this, the horizontal axis might represent time (τ), while the vertical axis represents the different positions along the string (σ). The surface you would sketch, showing how the string vibrates over time, is the worldsheet. The worldsheet can have different shapes and geometries depending on how the string is vibrating and interacting with other strings. These shapes are often described mathematically using surfaces like spheres, tori, and more complex geometries.

So the worldsheet is a surface that describes the history of a string in spacetime. For example, the worldsheet can be described by two parameters. Firstly, *σ (sigma).* This represents different points along the string (like its length).then we have *τ (tau).* This represents time, showing how the position of the string changes over time. In physics, the action is a quantity that summarizes the dynamics of a system. For strings, the action is typically defined by the Nambu-Goto action, which measures the area of the worldsheet. The less area the worldsheet covers, the more stable the string configuration is, which corresponds to physical principles like energy conservation.

As we told you before strings act differently according to different vibrations, when the string is at rest (not vibrating), it has a very simple worldsheet it would be just a straight line over time.

When there is something that causes a ripple in space it will cause the strings to vibrate, it will be like a ripple in water. These dynamics of the worldsheet are captured by the *Polyakov action*, a reformulation of another theory that is often more convenient for calculations.

$$\mathcal{S} = \frac{T}{2} \int \mathrm{d}^2\sigma \sqrt{-h} h^{ab} g_{\mu\nu}(X) \partial_a X^\mu(\sigma) \partial_b X^\nu(\sigma)$$

S: The action, a quantity used in the principle of least action to determine the path a system takes.

T: The string tension, a measure of the stiffness of the string. It characterizes how much energy is stored in the string when it is stretched.

$d\tau$ and $d\sigma$: The differential elements of the worldsheet coordinates, representing the time and position along the string.

h: The determinant of the worldsheet metric tensor, which describes the geometry of the worldsheet.

H^{ab}: The inverse of the worldsheet metric tensor, which relates to the geometry of the surface traced by the string in spacetime.

$g_{\mu\nu}$: The background spacetime metric tensor, which describes the geometry of the spacetime in which the string moves.

X_μ: The embedding coordinates of the string in spacetime, indicating the position of the string as a function of the worldsheet parameters σ\sigmaσ and τ\tauτ.

$\partial_a X^\mu$: The derivatives of the embedding coordinates with respect to the worldsheet parameters. These represent how the position of the string changes along the worldsheet.

The determinant $\sqrt{-h}$ accounts for the curvature and geometry of the worldsheet. It effectively measures the "area" of the worldsheet in a way that is independent of the specific coordinates chosen.

Its ability to incorporate geometrical aspects, facilitate interactions, and connect with quantum field theory makes it an indispensable tool for physicists exploring the fundamental nature of the universe through string theory. By using the Polyakov action, researchers can derive meaningful predictions and explore the implications of string theory in both particle physics and cosmology!

State Selection with GSO Projections.

We will tell this in short, state selection with GSO projections is a critical technique in string theory that serves to filter out unwanted states from the vast spectrum generated by string dynamics (like the tachyon we told you about earlier). By projecting states based on their properties (specifically using the fermion number operator), the GSO projection helps construct a physically viable theory, ensuring stability, maintaining supersymmetry, and eliminating non-physical states like tachyons. So state selection with GSO projections is a fundamental concept in string theory, (particularly in the context of type I and type II superstring theories). The GSO projection is essential for constructing physically viable string theories by selecting appropriate states from a larger set of possible states. Let's break down the concept step-by-step to understand what GSO projections are, why they are used, and their implications.

Dualities.

Dualities provide us from connections between certain theories, suggesting that what seem like separate models might actually be different perspectives of a single unified theory. A duality is a symmetry that tells us two different theories can have the same physical results, even if they look completely different in their setup or properties. We have multiple different dualities. One of these is *T-duality*, which shows that a string compactified on a small circular dimension behaves identically to one on a large circle, with large and small scales essentially interchangeable. This shows that strings can't resolve sizes smaller than this minimum, clashing with the classical idea of infinitely divisible space.

Another important duality is *S-duality,* which relates theories with strong interactions to those with weak interactions. This lets physicists switch to a simpler, weakly interacting version of a theory to study complex, strongly interacting systems. For instance, S-duality reveals that type I string theory with strong coupling is equivalent to heterotic SO(32) string theory with weak coupling, showing these seemingly distinct theories are in fact, the same.

Mirror symmetry is another duality used when string theory's extra dimensions are compactified on shapes called *Calabi-Yau manifolds (explained earlier)* . Mirror symmetry states that two geometrically different *Calabi-Yau spaces* can produce identical physical outcomes, opening up new mathematical tools for solving intricate geometry problems. *Uduality* combines aspects of both T- and S-duality and plays a significant role in M-theory and string theory.

D-branes.

D-branes are objects within string theory with specific dimensions that act as surfaces or boundaries where open strings can attach. For example:

D0-brane is a point-like object with zero dimensions.

D1-brane is a one-dimensional line or string.

D2-brane is a two-dimensional membrane or sheet, and so on.

a Dp-brane is a D-brane with p spatial dimensions. So, a D3-brane would be a 3 dimensional "volume" in spacetime, like a 3-dimensional surface in a higher dimensional space. In a theory that describes a universe with multiple spatial dimensions, D-branes can have dimensionalities that match those of the observable universe or extend into dimensions we can't see. D-branes enable the emergence of gauge fields, which describe forces in physics. The specific arrangement and interactions of D-branes can have gauge symmetries that correspond to known forces, such as the electromagnetic, weak, and strong nuclear forces.

Okey so there are a few ways that D-branes interact with the strings, Open strings have two endpoints, and in string theory with D-branes, these endpoints must be attached to a D-brane. The movement of these endpoints along the brane allows them to take on various vibrational modes, corresponding to particles that are constrained to the brane's dimensions. For example, if our observable universe were a D3-brane, particles like electrons would only exist within the three spatial dimensions of the brane. Logically speaking if we have open string we must also have closed strings… right? Of course we do! closed strings (which form loops) are not restricted to the branes and can move through the full higher-dimensional space. These strings are associated with gravity, as their vibrations include the graviton, the theoretical particle responsible for mediating gravitational force. Since gravitons aren’t restricted to D-branes, gravity can act across all dimensions, possibly explaining why gravity appears weaker than other forces.

D-branes can also play a role in producing *gauge symmetries*, which are fundamental to particle physics. When strings attach to D-branes, the vibrations of these strings can create fields along the brane's surface. Different types of D-brane configurations allow for different gauge groups (mathematical structures representing the symmetries of forces). These configurations can have symmetries similar to the *Standard Model*. This happens when multiple D-branes exist near each other, the strings can stretch between them, creating more complex vibrational patterns. These patterns correspond to different types of particles, allowing for particle structures similar to those seen in the *Standard Model*.

Representation theory and particle spectra.

Once the gauge groups are defined, *representation theory* becomes crucial in understanding the *particle spectra* of string theory. Particles are categorized by their representations of the gauge group, which dictates how they transform under different symmetries.

Representation theory is a theory that helps us understand how objects (like particles) can "represent" or embody the symmetries of a group. Think of it as a way of describing how different particles or systems respond to the symmetries described by a group. For example, if we know a system has certain symmetries the representation theory tells us how the components of that system should behave or transform when those symmetries are applied. The allowed representations tell us how many types of each particle can appear. This can naturally lead to families of particles, like the three generations of quarks and *leptons* we observe in nature. Each generation has similar properties but different masses. Representation theory explains why these particles have similar properties but appear in multiple "copies" or generations.

(Lectures on complex geometry, Calabi-Yau manifolds and toric geometry, sd) (ricci tensor, sd) (Dimensions in Mathematics: Definition, Types, Applications, and Examples, sd) (matrix introduction, sd) (understand the basics of matrices, sd) (fields in physics, sd) (different fields in physics, sd) (Introducing Conformal Field Theory, sd) (higgs boson, sd) (Lie groups, sd) (Does Conformal Invariance of the Polyakov Action in Conformal Gauge imply Conformal Invariance of the Pre-gauge-fixed Polyakov Action?, sd) (Calabi-Yau manifolds, 2023) (An Introduction to Calabi-Yau Manifolds, 2013) (calibi yau manifold, 2009) (D-branes and branes , sd) (leptons, sd) (zhelnorovich, 2016) (standart model, 2024) (standartmodel summary, 1998) (metric tensor, sd) (einsteins field equations, sd) (Dimensions in Mathematics: Definition, Types, Applications, and Examples, sd) (kaluza klein theory pdf, 2022) (electromagnetic field, sd) (SU 3 SU 2 U1, sd) (polynomials, sd) (Broken symmetry and the mass of gauge vector mesons, 1964) (Spontaneous symmetry breaking in gauge theories: a historical survey, 1998) (Broken symmetries and the masses of gauge bosons, 19 october 1964) (Field Quantization, scolar field, 1996) (An Introduction to Quantum Field Theory, scolar field, 1995) (Gauge Field Theories, vectors, higgs field,field lines, 2000)

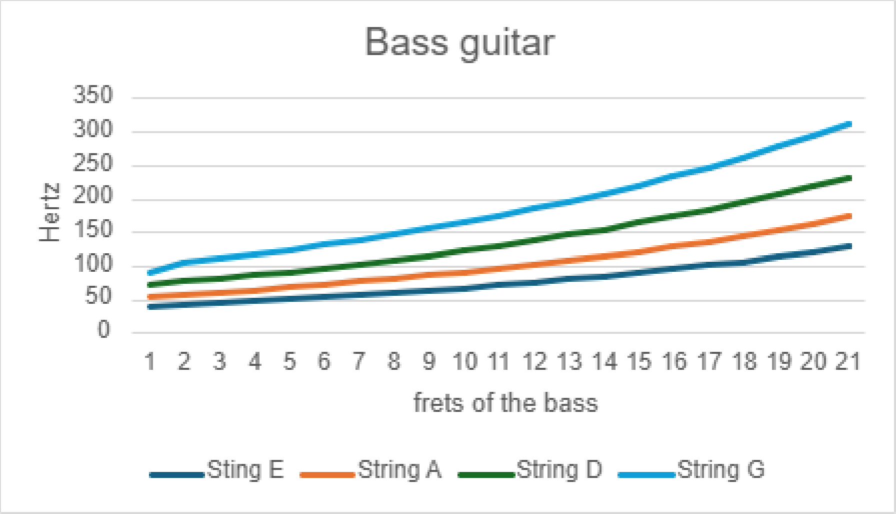
How does the frequency of a vibrating string change when adjusting the tension, thickness, or length, and how can these observations help us understand vibrations on a quantum level in string theory?

For this sub-question, will I do active research. I will be testing what Hertz a string makes if they are thicker, longer and have more tension. I will be doing this with rubber bands and my bass guitar. My hypothesis before my research is that a higher tension, a shorter length and a tinner string produce higher frequencies.
My study aims to understand what influences the vibrations of a string. I did this by finding different strings to study, in this way I was able to explore how these differences affect the vibration of a string. This helped me understand how vibrations work in string theory and how changes in the strings could affect their behavior at the quantum level.

Steps-by-step plan

1. I started out by gathering various strings. I chose 2 types of elastics, a small thin type of elastic and a big thick type of elastic.
2. I downloaded an app that was able to measure the hertz (frequencies) of the strings. I used 2 apps, the first is GuitarTuna, I used this app to tune my bass guitar, and the second is Tuner-Pitched, this helped me measure the hertz of the strings
3. I experimented with the tension, the thickness and the length of the strings. I would span them between 2 objects and would vibrate them and measure them.
4. I wrote down all the data and organized it, so it was readable.
5. The last step was to analyze all the data and determine how each factor affects the frequency.

The data that I got out of it is put in the table and graph right here. The first graph and table you see are from the bass guitar.



String E	String A	String D	String G
41	54,6	73,3	90,3
43,5	58,9	77,9	104,6
46,2	61,4	82,3	110,6
48,7	65,3	87,3	117,4
51,1	68,8	92	124,2
54,6	73,2	97,4	131,5
57,5	78	103,4	139,2
61	82,3	109,7	147,6
64,8	86,7	116	156,1
68,3	92,1	123,1	165,3
72,7	98,2	130,3	175,7
76,7	103,3	138,1	185,7
81,1	108,9	146,5	196,6
85,8	115,8	155,2	208,3
90,4	121,4	164,5	220,9
96,4	129	174	234,3
102,1	136	184,2	247,7
107	144,6	195,7	262,7
114,7	154,6	206,7	278,5
122,1	162,7	219,8	295,6
128,4	173,9	232,9	312,6

The next table is from the thin and small elastics.

cm	Hertz
Weak tension	
10	116,9
12,5	94,2
13	87,3
15,5	59
Medium tension	
10	136,7
12	94,6
13	86
15,5	68,4
Strong tension	
10	180,9
12	135,8
13	113,5
14,5	96,5

The next table was a little experiment i did with 2 strings that were almost the same length, they were the same thickness, but one had more tension than the other.

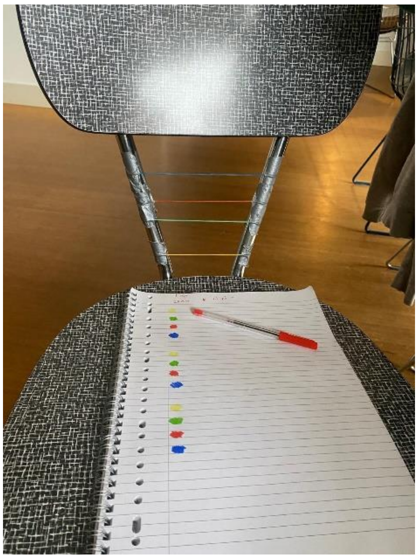
cm	Hertz
9,5	102,4
9,7	151,2

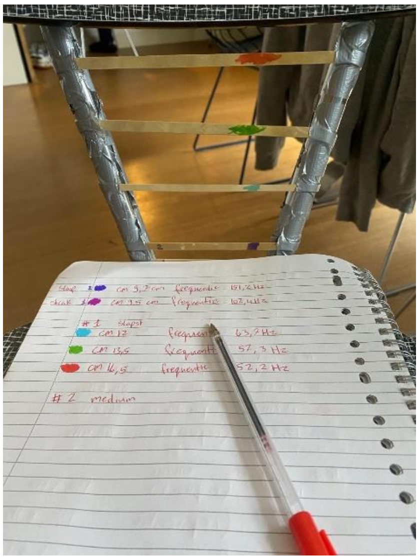
The upper one had a lot of tension, and the lower one had a little bit of tension.

The next table is the same as one before, it all the same elastic, it's a thick elastic, and the tension is different, and the length.

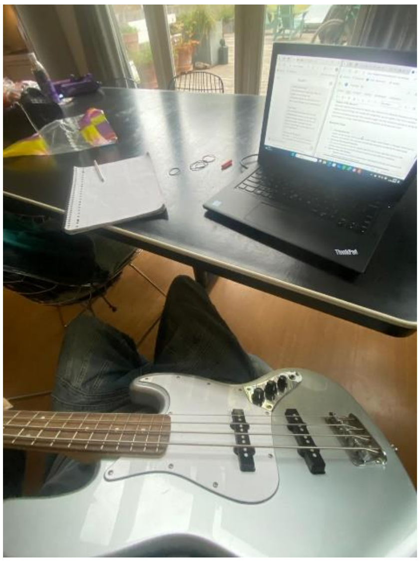
cm	Hertz
Weak tension	
12	63,7
13,5	57,3
16,5	52,2
Medium tension	
12,5	72,3
14,5	70,2
16,5	65,8
Strong tension	
12,5	95,7
14,5	84,4
17	71,5

Through this active research I found out that a longer string will have a lower frequency than a shorter string. A string with more tension will have a higher frequency and a thicker string will have a lower frequency. This matches my hypothesis. These observations might help illustrate how string theory works, just like with the research, the string in string theory vibrate with different frequencies based on different factors. For example, in my experiment, the difference in the thickness or length or tension of a string changes its frequency. This is the same om quantum level, different vibrations determine the properties of a particle, like mass or charge. Seeing these changes in bigger strings gives an example of how tiny vibrating quantum strings might influence the properties of particles in our universe.





slap 2 ● cm 9,7 cm frequentie 151,2 Hz
strak 1 ● cm 9,5 cm frequentie 102,4 Hz
1 slapst
● cm 12 frequentie 63,7 Hz
● cm 13,5 frequentie 57,3 Hz
● cm 16,5 frequentie 52,2 Hz
2 medium



Discussion

In the end we didn’t exactly predict how it all was connected to string theory but we did indeed guess that certain theories would come back in it because of Fitous pws light. ‘However there where things that I did not expect to be related to string theory in the aspects of math’s’- Fitou. Just like Merel would not have predicted that there where so many aspects of the strings that reflect in a guitar.

In making this work paper there were different types of challenges that have impact on the end result. The first one is the accuracy and reliability of the sources that we used. Most of the information that we found came from trusted sources and was backed up by other trustworthy sources, it was a big challenge to find the data that we needed and data that was relevant to the questions. Even though a big part of our sources are from universities or big physic sites that’s why we do have in mind that they all are pretty reliable.

Another difficult thing was understanding the concept of string theory and its dimensions. After the third dimension, it is really hard to imagine how a universe would look with these additional dimensions. It is even harder to explain these weird concepts. Another problem was trying to figure out what was too deep into the content and what we did have to explain and what we didn’t, this was especially difficult for fitou while making the mathematics sub-question because of its difficult math’s.

This theory is still a theory as well, this means that it could be proven wrong. It can change a lot in a short period of time, this means that the information we have could change. This theory can be way different in a short period of time, or it could stay the same and can be proven right any time soon.

Summed up (samenvatting)

String theory is a theoretical framework in physics proposing that the building blocks of the universe are not point particles but one-dimensional "strings" that vibrate at specific frequencies. These vibrations define the properties of particles, such as mass and charge, and aim to unify all fundamental forces, including gravity, electromagnetism, and the strong and weak nuclear forces, into a single framework.

The theory introduces the concept of additional dimensions beyond the observable four (three spatial and one temporal). In string theory, the universe consists of 10 dimensions, while M-theory extends this to 11. These extra dimensions are hypothesized to be "compactified" or curled up into tiny spaces, often modeled using Calabi-Yau manifolds. These geometric shapes influence the vibrational modes of strings and, therefore, the physical characteristics of particles.

Mathematics plays a critical role in string theory. Important theories and factors include Ricci curvature, which measures the bending of space. Calabi-Yau manifolds, which stabilize the compactified dimensions, and group theory, which describes the symmetries of particle interactions. Matrices and algebra are also used to manage relationships in higher-dimensional spaces, allowing for precise modeling of string behaviors.

Vibrations of strings are central to the theory, with different vibrational modes corresponding to distinct particles. The Higgs field, described as a scalar field, interacts with particles, giving them mass through a process analogous to resistance in a medium. Supersymmetry (SUSY) is another integral aspect, proposing a corresponding partner particle for every known particle, which resolves inconsistencies in the mathematical model, though SUSY remains experimentally unconfirmed.

String theory also offers explanations for the locations and effects of extra dimensions. Compactification suggests these dimensions are so small they are undetectable, while the theory of large extra dimensions posits that gravity's relative weakness arises from its spread across these dimensions. Additionally, the holographic principle theorizes that our three-dimensional universe could be a projection of information stored on a two-dimensional surface.

Although string theory remains unproven and is for a big part completely theoretical, it provides a promising framework for reconciling quantum mechanics with general relativity. It offers insights into things such as black holes, the Big Bang, and even the multiverse, So that’s why it is one of the most important and interesting theories that we have made till now.

Conclusion

String theory represents one of the most successful attempts in modern physics to unify our understanding of the universe. Now to fully understand our head question we will break it down to the different subjects that have to do with string theory and to what extent they have to do with it and its dimension. And if you didn’t get it the ultimate answer to our head question is that the relationship between string theory, dimensions, and mathematics is profound and inseparable. Dimensions provide the framework within which the theory operates, while mathematics serves as the ‘language’ that describes its intricacies. Together, they allow for string theory to make sense in a way no other theories have yet to do, from the nature of particles and forces to the structure of the universe and the unification of quantum mechanics and gravity, it all adds up. Through its intricate interplay of higher-dimensional spaces and advanced mathematics, string theory continues to push the boundaries of our understanding of the cosmos. So let’s break it down.

The Role of Dimensions in String Theory

The concept of dimensions is one of the biggest parts of string theory. In our everyday experience, we are familiar with three spatial dimensions and one dimension of time, collectively known as spacetime. However, string theory extends this framework by requiring additional spatial dimensions to function. The most studied version of string theory uses ten dimensions, while the related M-theory extends this to eleven. These extra dimensions are not directly observable, instead, they are thought to be "compactified," curled up so tightly that they exist on scales smaller than can currently be detected.

One way to visualize compactified dimensions is through the analogy of an ant walking along a straw. To the ant, the straw has two dimensions: it can move along its length and around its circumference. However, when viewed from a distance, the straw appears as a one-dimensional line because its second dimension (the circumference) is too small to perceive. Similarly, the extra dimensions in string theory are believed to be compactified into intricate shapes, often described mathematically as Calabi-Yau manifolds. These complex geometrical spaces allow the extra dimensions to exist without interfering with the familiar four-dimensional spacetime we experience.

These additional dimensions are not merely theoretical and for fun, they are essential to the mathematical consistency of string theory. For example, without these extra dimensions, string theory cannot account for certain quantum inconsistencies or predict the existence of particles like the graviton, which mediates the force of gravity. Further, these dimensions play a role in explaining phenomena like the unification of forces, as they allow for interactions that cannot occur in lower-dimensional spaces.

Mathematics: The Language of String Theory

So mathematics serves as the ‘’language’’ through which the multi-dimensional nature of string theory is articulated. Differential geometry, algebra, group theory, and tensor calculus are indispensable tools for understanding and formulating the theory. One of the key mathematical requirements in string theory is the concept of Ricci-flat spaces. These are spaces where the Ricci curvature tensor equals zero (Rmn=0), indicating a balance in curvature that allows compactified dimensions to remain stable without disrupting observable physics. Ricci-flatness is critical in defining the geometry of the extra dimensions and ensuring the theory's consistency.

Another significant mathematical structure in string theory is group theory, which organizes symmetries and transformations of particles and forces. The symmetry groups SU(3), SU(2), and U(1) describe the Standard Model of particle physics, which includes the strong nuclear force, weak nuclear force, and electromagnetism. In string theory, these symmetry groups extend to describe interactions in higher-dimensional spaces, providing a framework for understanding how strings vibrate and interact.

Matrices are another critical mathematical tool used in string theory. They help describe transformations and interactions in multi-dimensional spaces, organizing the relationships between particles, forces, and dimensions. For instance, the mixing matrices used in the Standard Model encode the probabilities of particles transforming into one another and help physicists understand the patterns of particle interactions. Matrices can also represent vibrational modes of strings, so it's like linking their frequencies and amplitudes to the properties of particles.

Connecting Vibrations to Physics (and use of worldsheet)

In string theory, the vibrations of strings determine the properties of particles. (also shown in the practical experiment) Different vibrational modes correspond to different particles, with the frequency and pattern of vibration dictating properties such as mass and charge. These vibrations are mathematically described using tools from quantum mechanics and field theory. For instance, the behavior of a string in spacetime is mapped using a "worldsheet," a two-dimensional surface that traces the string's movement over time. The dynamics of the worldsheet are described by actions like the *Polyakov* action, which encodes the energy and geometry of the string's movement. The connection between string vibrations and particle properties also links string theory to the broader framework of quantum field theory. Fields such as the Higgs field, which provides particles with mass, are viewed in string theory as manifestations of specific vibrational modes. This perspective helps bridge the gap between the Standard Model and the deeper, more fundamental structures predicted by string theory.

Dimensions, Gravity, and the Universe

One of the key motivations for string theory is its potential to unify quantum mechanics and general relativity, two foundational theories of physics that are currently incompatible. The extra dimensions of string theory provide a framework for describing quantum gravity, a goal in theoretical physics. For example, the graviton, the hypothetical quantum particle that mediates gravity, naturally uses the vibrations of strings in higher-dimensional spaces. Beyond quantum gravity, the dimensions of string theory have implications for understanding the nature of the universe itself. As an example, the theory suggests that gravity might appear weaker than other forces because it can "leak" into the extra dimensions. This idea provides a potential explanation for the hierarchy problem, which then explains why gravity is so much weaker than the other forces.

String theory also gives insight into the concept of the multiverse. The arrangement of compactified dimensions could lead to a vast number of possible universes, each with its own set of physical laws and constants. This diversity arises from the countless ways in which the extra dimensions can be compactified, with each configuration corresponding to a different universe.

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Reflection

Okey so, there were a lot of things that actually went very well. Fitou loved explaining stuff to Merel and we both enjoyed talking and researching about this subject. Fitou had started very early on with writing and kept a very good clear view of what she had to do and what was left, and for one time she actually kept her word documents clean with not text all over the place. Merel didn't know a lot about the subject and kept learning about stuff so she got a really good view about everything honestly. One thing that went a bit less great was Merel's timing she began a bit late and Fitou kept pressing her which was annoying and stressing for both of us.

The teamwork had its ups and downs, sometimes there was a great communication, other times there was no communication. It worked out, but it could have been better. The planning and the task distribution could have been better and more of a plan and less of a big mess. The planning was okay for the both of us, it could have been better for Merel, she needed to start earlier, and Fitou her planning was good, she started really early.

Merel did not know a lot about string theory. So everything was new even the basics where pretty new so she learned a lot from this and enjoyed learning about it! Fitou surprisingly also learned a lot about this subject even though she knew a lot already, she knew more about the theories than the math that's why it was fun to learn something new!

If we would to redo it we think we would do the planning differently and have a different active research, we initially wanted to interview a professor at a university however we failed to get a connection with them and so failed to do this. So that's something that definitely would be different.

We would probably continue with the further versions of string theory like M theory that has 11 dimensions and go further into the difficult math's.

We thank you for reading out PWS and hope you gained some knowledge about the amazing string theory!

Fitou's logboek!

	wat	hoelang	minuten
26-4-2024	onderzoek doen 1ste deelvraag what is string theory	2 uur en 36 min	156
28-4-2024	klein deel schrijven (schets) van wat ik wil in deelvraag	1 uur en 20 min	80
29-4-2024	filmpjes kijken over string theory	1 uur en 23 min	83
29-4-2024	stukje verbeteren en schrijven deelvraag 1	1 uur 48 min	108
30-4-2024	procedure schrijven	15 min	15
3-5-2024	onderzoeken en schrijven over dimensies	49 min	49
4-5-2024	'afmaken'' deelvraag 1	3 uur en 4 min	184
27-5-2024	bronnen voor merel zoeken	20 tot 30 minuten	30
27-5-2024	bronnen lezen en info opzoeken over dimensies	50 minuten	50
3-6-2024	nadenken over volgende deelvragen (tijdens b uur	1 uur	60
4-6-2024	pws deel1 afmaken (hypotese maken)	17 min	17
4-6-2024	opmaak pws deel 1	39 min	39
11-6-2024	afmaken pws deel 1	30 min	30
12-6-2024	inleveren pws deel 1	_	5
2-7-2024	deelvraag 1 verbeteren	20 min	20
12-7-2024	deelvraag 1 af maken	40 min	40
12-7-2024	begrippen deelvraag 1 uileggen in theoretisch kader	50 min	50
3-8-2024	merel bellen om uitleg te geven	22 min	22
6-8-2024	extra deelvragen bedenken en deeltjes uitwerken	1 uur en 41 min	101
22-8-2024	lezen over 2e deelvraag dimensies	50 min	50
23-8-2024	sdeel van deelvraag 2 schrijven (1ste tot 5de dimensie)	3 uur 20 min	200
26-8-2024	6de dimensie geschreven en onderozzke doen compact D	1 uur	60
29-8-2024	pws ordenen en afschrijven 2e deelvraag	4 uur	240
30-8-2024	actief onderzoek bedenken en merel uitleg geven	5 uur	300
8-9-2024	theoretisch kader schrijven van deelvraag 2	30 min	30
8-9-2024	b uur marc bespreken pws	15 min	15
21-9-2024	actief onderzoek en pws markt bedenken	20 min	20
21-9-2024	deelvraag 2 afgerond en uitleg vids kijken over matrices	48 min	48
29-9-2024	D-branes uitleg bekijken+theoretisch kaderer van afmaken	1 uur 4 min	64
2-10-2024	beginnen deelvraag mathmatics (wat moet er in komen enzo)	1 uur en 48 min	104
9-10-2024	beginnen met ricci curvature	3 uur 27 min	207
12-10-2024	calibi cau manifolds & kazula klein theory	20 min	20
13-10-2024	verder aan calibi + einsteins field equatations	1 uur 53 min	113
14-10-2024	deelvraag 2 dingen aanpassen en beter uitleggen	1 uur	60
15-10-2024	films kijken over string theory en lezen	4 uur en 57 min	297
17-10-2024	deelvraag 1 verbeteren en meer schrijven	40 min	40
24-10-2024	algerbra	3 uur	180
24-10-2024	group theory	1 uur	60
25-10-2024	more group theory and ricci curvature!	4 uur	240
25-10-2024	theoretisch kader vectors onderzoek doen	2 uurtjes(meer?)	130
26-10-2024	merel helpen en aanmoedingen om te beginnen	26 min	26
26-10-2024	beginnen aan matrices, uitleg kijken	3 uur en 45 min	225
27-10-2024	uitzoeken wtf fields zijn	1 uur 30 min	90
28-10-2024	struggelen met actief onderzoek :(	20 min	20
28-10-2024	schrijven over fields en gauge theory	3 uur	180

30-10-2024	schrijven over Higgs en theoretisch kader aanpassen	1 uur 10 min	70
30-10-2024	schrijven over verschillende fields	2 uur	120
31-10-2024	compactifivation beter uitwerken	40 min	40
1-11-2024	worksheet dynamics en dualities	2 uue	120
2-11-2024	workshet dynamics wiskunde proberen te snappen	3 uur	180
3-11-2024	dualities en verder met d-branes	4 uur en 30 min	270
3-11-2024	particle spectra	40 min	40
4-11-2024	bronnen opschrijven en invoegen	1 uur	60
4-11-2024	theoretisch kader schrijven over deze deelvraag	2 uur en 20 min	140
5-11-2024	afmaken deelvraag	3 uur	180
6-11-2024	beginnen conclusie	20 min	20
8-11-2024	opnieuwmaken wat was verwijdert....	3 uur + 1 uur(Wb uur)	240
13-11-2024	conclusie schrijven	1 uur	60
13-11-2024	spelling checken en merel haar stukken in erin zetten	1 uur	60
20-11-2024	samenvatting maken van pws	20 min	20
totaal:		96 uur en 30 min	5778

Merel's Logboek

Samen 1 uur 11-4 (uitleg + nadenken wat)

school Samen 45 min 12-4 (welk onderwerp)

school Samen 1 uur 20 min 16-5 (kick off)

school Merel 2 uur 27-5 (onderzoeksplan + deelvragen)

school Merel 3 uur 30-5 (informatie)

school Merel 45 min 31-5 (informatie)

school Merel 3 uur en 30 min 21-7 (informatie)

thuis Merel 2 uur 30 min 27-7 (info (boek lezen))

vakantie (spanje) Merel 3 uur 19 min 5-8 (info (boek lezen))

vakantie (portugal) Merel 3 uur 3 min 6-8 (info (boek lezen))

vakantie (portugal) Samen 23 min 6-8 (bespreken/taakverdeling)

vakantie (potugal) Merel 1 uur en 30 min 15-8 (lezen/deelvraag)

vakantie (frankrijk) Merel 3 uur 29-8 (pich en info en lezen)

school Merel 1 uur en 30 min 29-8 (info)

thuis Samen 5 uur 30-8 (info, deelvragen en logboek/organiseren)

school Merel 2 uur 2-9 (info)

school Merel 1 uur 9-9 (info)

school Merel 4 uur en 45 min 9-9 (deelvragen)

thuis Merel 1 uur en 30 min 18-9 (deelvraag) thuis Merel 4 uur en 20 min 29-10 (deelvraag maken)

thuis Merel 4 uur en 30 min 30-10 (deelvraag maken)

thuis Merel 5 uur 10 min 31-10 (deelvragen maken)

thuis Merel 6 uur 1-11 (actief onderzoek)

thuis Merel 3 uur en 25 min 2-11 (deelvragen)

thuis Merel 2 uur 4-11 (opmaak en logboek en deelvraag)

school Merel 1 uur 5-11 (deelvraag)

school Samen 30 min 12-11 (feedback)

school Merel 1 uur 50 min 19-11 (discussie en pws markt nadenken)

school en in bieb Merel 20 min 20-11 (laatste check)

Merel: deelvragen 3,5 en 6 en discussie.

Fitou: deelvragen 1,2 en 4 + conclusie, reflectie, theoretisch kader, opmaak, samenvatting.

Samen: procedure, hypothese.