UITWERKINGEN DEEL 1 | WISKUNDE | 12E EDITIE | 3 HAVO

GETAL & RUIMTE

Noordhoff Uitgevers

5 Algebraïsche vaardigheden

Voorkennis Haakjes, breuken en wortels

Bladzijde 166

1
a $x(2x-5) = 2x^2 - 5x$
b $3x(x+8) = 3x^2 + 24x$
c $(x-1)(x+6) = x^2 + 6x - x - 6 = x^2 + 5x - 6$
d $(x+5)(x-5) = x^2 - 5x + 5x - 25 = x^2 - 25$
e $(x+7)(x+7) = x^2 + 7x + 7x + 49 = x^2 + 14x + 49$
f $-x(x+7) = -x^2 - 7x$
g $(2x-1)(x+7) = 2x^2 + 14x - x - 7 = 2x^2 + 13x - 7$
h $-9x(x-1) = -9x^2 + 9x$
i $(x-5)(y-2) = xy - 2x - 5y + 10$

2
a $4(a+2) + 3a = 4a + 8 + 3a = 7a + 8$
b $4a + 2(a+3) = 4a + 2a + 6 = 6a + 6$
c $4a - 2(a+3) = 4a - 2a - 6 = 2a - 6$
d $(4a+2)(a+3) = 4a^2 + 12a + 2a + 6 = 4a^2 + 14a + 6$
e $(4a-2)(a+3) = 4a^2 + 12a - 2a - 6 = 4a^2 + 10a - 6$
f $3(a-2) - 5a = 3a - 6 - 5a = -2a - 6$
g $3a - 2(a-5) = 3a - 2a + 10 = a + 10$
h $(3a+2)(a-5) = 3a^2 - 15a + 2a - 10 = 3a^2 - 13a - 10$
i $(3a-2)(a-5) = 3a^2 - 15a - 2a + 10 = 3a^2 - 17a + 10$

Bladzijde 167

3
a $(a+4)(a-4) = a^2 - 16$
b $(x-6)(x+6) = x^2 - 36$
c $(3a+2)(3a-2) = 9a^2 - 4$
d $(5y-1)(5y+1) = 25y^2 - 1$
e $(2a+b)(2a-b) = 4a^2 - b^2$
f $(3m-5n)(3m+5n) = 9m^2 - 25n^2$

4
a $(x+4)^2 = x^2 + 8x + 16$
b $(b-6)^2 = b^2 - 12b + 36$
c $(2c+9)^2 = 4c^2 + 36c + 81$
d $(3p-7)^2 = 9p^2 - 42p + 49$
e $(5a+x)^2 = 25a^2 + 10ax + x^2$
f $(4v-5z)^2 = 16v^2 - 40vz + 25z^2$

5
a $(3x+8)^2 = 9x^2 + 48x + 64$
b $(8-3x)^2 = 64 - 48x + 9x^2$
c $(8x+3)(8x-3) = 64x^2 - 9$
d $(5a-z)(5a+z) = 25a^2 - z^2$
e $(6b-5k)^2 = 36b^2 - 60bk + 25k^2$
f $(4x+11y)(4x-11y) = 16x^2 - 121y^2$

Bladzijde 168

6
a $\frac{1}{5} + \frac{3}{4} = \frac{4}{20} + \frac{15}{20} = \frac{19}{20}$
b $\frac{1}{5} \cdot \frac{3}{4} = \frac{3}{20}$
c $\frac{1}{5} : \frac{3}{4} = \frac{1}{5} \cdot \frac{4}{3} = \frac{4}{15}$
d $3\frac{1}{2} + \frac{3}{4} = \frac{7}{2} + \frac{3}{4} = \frac{14}{4} + \frac{3}{4} = \frac{17}{4} = 4\frac{1}{4}$
e $3\frac{1}{2} \cdot \frac{3}{4} = \frac{7}{2} \cdot \frac{3}{4} = \frac{21}{8} = 2\frac{5}{8}$
f $3\frac{1}{2} : \frac{3}{4} = \frac{7}{2} \cdot \frac{4}{3} = \frac{28}{6} = \frac{14}{3} = 4\frac{2}{3}$
g $1\frac{2}{3} + 2\frac{1}{2} = \frac{5}{3} + \frac{5}{2} = \frac{10}{6} + \frac{15}{6} = \frac{25}{6} = 4\frac{1}{6}$
h $1\frac{2}{3} \cdot 2\frac{1}{2} = \frac{5}{3} \cdot \frac{5}{2} = \frac{25}{6} = 4\frac{1}{6}$
i $1\frac{2}{3} : 2\frac{1}{2} = \frac{5}{3} : \frac{5}{2} = \frac{5}{3} \cdot \frac{2}{5} = \frac{10}{15} = \frac{2}{3}$

7
a $3\frac{1}{4} - 2\frac{1}{2} = \frac{13}{4} - \frac{5}{2} = \frac{13}{4} - \frac{10}{4} = \frac{3}{4}$
b $3\frac{1}{4} \cdot 2\frac{1}{2} = \frac{13}{4} \cdot \frac{5}{2} = \frac{65}{8} = 8\frac{1}{8}$
c $3\frac{1}{4} : 2\frac{1}{2} = \frac{13}{4} : \frac{5}{2} = \frac{13}{4} \cdot \frac{2}{5} = \frac{26}{20} = \frac{13}{10} = 1\frac{3}{10}$
d $2 - \frac{2}{3} = 1\frac{1}{3}$
e $2 \cdot \frac{2}{3} = \frac{4}{3} = 1\frac{1}{3}$
f $2 : \frac{2}{3} = 2 \cdot \frac{3}{2} = \frac{6}{2} = 3$
g $\frac{2}{3} - 2 = -1\frac{1}{3}$
h $\frac{2}{3} \cdot 2 = \frac{4}{3} = 1\frac{1}{3}$
i $\frac{2}{3} : 2 = \frac{2}{3} \cdot \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$

8 **a** $5\sqrt{2} \cdot 3\sqrt{5} = 15\sqrt{10}$
b $3\sqrt{6} \cdot 2\sqrt{5} = 6\sqrt{30}$
c $5\sqrt{2} \cdot 3\sqrt{2} = 15\sqrt{4} = 15 \cdot 2 = 30$
d $5\sqrt{3} \cdot 7\sqrt{12} = 35\sqrt{36} = 35 \cdot 6 = 210$
e $5\sqrt{9} \cdot 7 = 5 \cdot 3 \cdot 7 = 105$
f $3\sqrt{6} \cdot -2\sqrt{6} = -6\sqrt{36} = -6 \cdot 6 = -36$

9 **a** $5\sqrt{2} + 8\sqrt{2} = 13\sqrt{2}$
b $5\sqrt{2} - \sqrt{2} = 4\sqrt{2}$
c $3\sqrt{5} - 7\sqrt{5} = -4\sqrt{5}$
d $\sqrt{7} + \sqrt{7} = 2\sqrt{7}$
e $\sqrt{36} - \sqrt{9} = 6 - 3 = 3$
f $2\sqrt{3} + 3\sqrt{2}$ kan niet

10 **a** $3\sqrt{2} + 5\sqrt{7}$ kan niet
b $3\sqrt{2} \cdot 5\sqrt{7} = 15\sqrt{14}$
c $5\sqrt{7} \cdot 2\sqrt{7} = 10\sqrt{49} = 10 \cdot 7 = 70$
d $5\sqrt{7} - 2\sqrt{7} = 3\sqrt{7}$
e $3\sqrt{25} - 5\sqrt{9} = 3 \cdot 5 - 5 \cdot 3 = 15 - 15 = 0$
f $9\sqrt{2} \cdot -3\sqrt{8} = -27\sqrt{16} = -27 \cdot 4 = -108$

5.1 Haakjes wegwerken

Bladzijde 169

1 **a** $(3x)^2 = 3^2 \cdot x^2 = 9x^2$
b $-p + 3(p - 4) = -p + 3p - 12 = 2p - 12$
c $(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$

2 **a** $3(x + 1)(x + 4) = 3(x^2 + 4x + x + 4) = 3(x^2 + 5x + 4) = 3x^2 + 15x + 12$
b $-2(a + 3)(a - 5) = -2(a^2 - 5a + 3a - 15) = -2(a^2 - 2a - 15) = -2a^2 + 4a + 30$
c $(5x)^2 + 4x(1 - 6x) = 25x^2 + 4x - 24x^2 = x^2 + 4x$
d $(2y)^2 - 4y(y - 5) = 4y^2 - 4y^2 + 20y = 20y$
e $3(2d + 1)(4 + d) = 3(8d + 2d^2 + 4 + d) = 3(9d + 2d^2 + 4) = 27d + 6d^2 + 12$
f $-3(6x - 1)(1 + 2x) = -3(6x + 12x^2 - 1 - 2x) = -3(4x + 12x^2 - 1) = -12x - 36x^2 + 3$
g $2(x + 3)(x - 5) = 2(x^2 - 5x + 3x - 15) = 2(x^2 - 2x - 15) = 2x^2 - 4x - 30$
h $5(n + 2)(n - 1) - 5n^2 = 5(n^2 - n + 2n - 2) - 5n^2 = 5n^2 - 5n + 10n - 10 - 5n^2 = 5n - 10$

3 **a** $6(a + 5)(2a - 1) = 6(2a^2 - a + 10a - 5) = 6(2a^2 + 9a - 5) = 12a^2 + 54a - 30$
b $-4(1 + 3x)(3x - 1) = -4(3x - 1 + 9x^2 - 3x) = -4(-1 + 9x^2) = 4 - 36x^2$
c $(5c)^2 + 4(2c - 3)(c + 1) = 25c^2 + 4(2c^2 + 2c - 3c - 3) = 25c^2 + 8c^2 + 8c - 12c - 12 = 33c^2 - 4c - 12$
d $\frac{1}{2}(5e - 1)(e + 6) = \frac{1}{2}(5e^2 + 30e - e - 6) = \frac{1}{2}(5e^2 + 29e - 6) = 2\frac{1}{2}e^2 + 14\frac{1}{2}e - 3$
e $8k + 2(4k - 1)(k + 4) = 8k + 2(4k^2 + 16k - k - 4) = 8k + 8k^2 + 32k - 2k - 8 = 38k + 8k^2 - 8$
f $2(a + 2)(5 - a) - (3a)^2 = 2(5a - a^2 + 10 - 2a) - 9a^2 = 10a - 2a^2 + 20 - 4a - 9a^2 = 6a - 11a^2 + 20$

4 $(3x)^2 = 3^2 \cdot x^2 = 9x^2$, dus $4(3x)^2 = 4 \cdot 9x^2 = 36x^2$.
Dus Bob heeft gelijk.

Bladzijde 170

5 **a** $3(x + 5)(x - 5) = 3(x^2 - 25) = 3x^2 - 75$
b $-4(a + 1)^2 = -4(a^2 + 2a + 1) = -4a^2 - 8a - 4$
c $2(c - 6)^2 = 2(c^2 - 12c + 36) = 2c^2 - 24c + 72$
d $5(2y - 1)^2 = 5(4y^2 - 4y + 1) = 20y^2 - 20y + 5$
e $-3(6 - e)(6 + e) = -3(36 - e^2) = -108 + 3e^2$
f $\frac{1}{2}(3x - 2)^2 = \frac{1}{2}(9x^2 - 12x + 4) = 4\frac{1}{2}x^2 - 6x + 2$

6 **a** $5(3x - 1)^2 = 5(9x^2 - 6x + 1) = 45x^2 - 30x + 5$
b $-(6x - 1)(6x + 1) = -(36x^2 - 1) = -36x^2 + 1$
c $5(2a - b)^2 = 5(4a^2 - 4ab + b^2) = 20a^2 - 20ab + 5b^2$
d $3(n + 2)^2 - 6(n - 1) = 3(n^2 + 4n + 4) - 6n + 6 = 3n^2 + 12n + 12 - 6n + 6 = 3n^2 + 6n + 18$
e $-4(b + 5)(b - 5) + (2b)^2 = -4(b^2 - 25) + 4b^2 = -4b^2 + 100 + 4b^2 = 100$
f $(1 - 4p)^2 - 8p(2p + 1) = 1 - 8p + 16p^2 - 16p^2 - 8p = 1 - 16p$

7 **a** $6p(3p-2)-(p+5)(p-4)=18p^2-12p-(p^2-4p+5p-20)=18p^2-12p-p^2+4p-5p+20=17p^2-13p+20$

b $(5p+1)^2-(3p+2)(p-7)=25p^2+10p+1-(3p^2-21p+2p-14)=25p^2+10p+1-3p^2+21p-2p+14=22p^2+29p+15$

c $(2x-3)^2-9(x-1)(x-8)=4x^2-12x+9-9(x^2-8x-x+8)=4x^2-12x+9-9x^2+72x+9x-72=-5x^2+69x-63$

d $(3x)^2-3(x+5)(x-2)=9x^2-3(x^2-2x+5x-10)=9x^2-3x^2+6x-15x+30=6x^2-9x+30$

e $(4x-3)(5x+8)-(3x-2)^2=20x^2+32x-15x-24-(9x^2-12x+4)=20x^2+32x-15x-24-9x^2+12x-4=11x^2+29x-28$

f $(5p)^2-(5p-3)^2=25p^2-(25p^2-30p+9)=25p^2-25p^2+30p-9=30p-9$

g $(5-x)^2-5(x+5)^2=25-10x+x^2-5(x^2+10x+25)=25-10x+x^2-5x^2-50x-125=-100-60x-4x^2$

h $(2p+5)^2-(2p-5)^2=4p^2+20p+25-(4p^2-20p+25)=4p^2+20p+25-4p^2+20p-25=40p$

8 **a** $A=(3n+7)^2-6(7n+8)(7n-8)$
$A=9n^2+42n+49-6(49n^2-64)$
$A=9n^2+42n+49-294n^2+384$
$A=-285n^2+42n+433$

b $B=5t(t-5)^2+(5-t)^2$
$B=5t(t^2-10t+25)+25-10t+t^2$
$B=5t^3-50t^2+125t+25-10t+t^2$
$B=5t^3-49t^2+115t+25$

c $C=(5t)^2-3(2t-4)^2$
$C=25t^2-3(4t^2-16t+16)$
$C=25t^2-12t^2+48t-48$
$C=13t^2+48t-48$

d $D=12pq+2p(q-3)^2$
$D=12pq+2p(q^2-6q+9)$
$D=12pq+2pq^2-12pq+18p$
$D=2pq^2+18p$

9 **a** $55\cdot 65=(60-5)(60+5)=3600-25=3575$

b $2\frac{1}{6}\cdot 1\frac{5}{6}=(2+\frac{1}{6})(2-\frac{1}{6})=4-\frac{1}{36}=3\frac{35}{36}$

c $101^2-99^2=(101+99)(101-99)=200\cdot 2=400$

d $10\cdot 99^2=10(100-1)^2=10(100^2-2\cdot 100\cdot 1+1^2)=10(10\,000-200+1)=10\cdot 9801=98\,010$

5.2 Breuken herleiden

Bladzijde 171

10 **a** $\frac{3p}{7p}=\frac{3}{7}$ **b** $\frac{xy}{xz}=\frac{y}{z}$ **c** $\frac{14a}{20a}=\frac{7}{10}$

11 **a** $\frac{-24xz}{30xyz}=-\frac{4}{5y}$

b $-\frac{36abc}{18ac}+\frac{9bc}{3c}=-2b+3b=b$

c $\frac{25pqs}{5p}-\frac{64qrs}{8r}=5qs-8qs=-3qs$

12 **a** $\frac{5b}{13b}=\frac{5}{13}$ **b** $\frac{5b}{15a}=\frac{b}{3a}$ **c** $\frac{25a}{a}=25$ **d** $\frac{a}{25a}=\frac{1}{25}$ **e** $\frac{18xy}{24xz}=\frac{3y}{4z}$ **f** $\frac{-21ab}{35abc}=-\frac{3}{5c}$ **g** $\frac{-20pqr}{25prs}=-\frac{4q}{5s}$ **h** $\frac{17abc}{17ab}=c$ **i** $\frac{17ab}{17abc}=\frac{1}{c}$

13 **a** $\frac{18xy}{6x} - 5y = 3y - 5y = -2y$

b $\frac{-27xy}{9y} + 5x = -3x + 5x = 2x$

c $-\frac{8ab}{2b} + \frac{9ac}{3c} = -4a + 3a = -a$

Bladzijde 172

14 **a** $\frac{6abc}{9a} = \frac{2bc}{3} = \frac{2}{3}bc$ **b** $\frac{49pqr}{14qr} = \frac{7p}{2} = 3\frac{1}{2}p$ **c** $\frac{72xyz}{-6xz} = -12y$

15 **a** $\frac{7p}{28} = \frac{p}{4} = \frac{1}{4}p$ **b** $\frac{15a}{45} = \frac{a}{3} = \frac{1}{3}a$ **c** $\frac{21x}{35} = \frac{3x}{5} = \frac{3}{5}x$ **d** $\frac{6q}{3} = 2q$ **e** $\frac{8c}{-8} = -c$ **f** $\frac{8y}{16} = \frac{y}{2} = \frac{1}{2}y$

16 **a** $\frac{4xy}{8x} = \frac{y}{2} = \frac{1}{2}y$ **b** $\frac{6abc}{2ab} = 3c$ **c** $\frac{p}{7p} = \frac{1}{7}$ **d** $\frac{6pq}{3q} = 2p$ **e** $\frac{3xy}{-6x} = \frac{y}{-2} = -\frac{1}{2}y$ **f** $\frac{12abc}{-4ac} = -3b$

Bladzijde 173

17 **a** $2a - \frac{3ab}{8b} = 2a - \frac{3a}{8} = 2a - \frac{3}{8}a = 1\frac{5}{8}a$

b $\frac{3}{4}x + \frac{5xy}{10y} = \frac{3}{4}x + \frac{x}{2} = \frac{3}{4}x + \frac{1}{2}x = \frac{3}{4}x + \frac{2}{4}x = \frac{5}{4}x = 1\frac{1}{4}x$

c $\frac{3abc}{ac} - \frac{6ab}{20a} = 3b - \frac{3b}{10} = 3b - \frac{3}{10}b = 2\frac{7}{10}b$

18 **a** $\frac{a(b+c)}{6(b+c)} = \frac{a}{6} = \frac{1}{6}a$ **b** $\frac{4(p+q)}{2(p+q)} = 2$ **c** $\frac{xy}{x(x+y)} = \frac{y}{x+y}$

19 $\frac{xy + 2x}{xy} = \frac{x(y+2)}{xy} = \frac{y+2}{y}$

Dus Hind heeft gelijk.

Bladzijde 174

20 **a** $\frac{8a^2 - 16a}{4a} = \frac{4a(2a-4)}{4a} = 2a - 4$

b $\frac{p+q}{5p+5q} = \frac{p+q}{5(p+q)} = \frac{1}{5}$

c $\frac{a^2 + 2a - 3}{2a+6} = \frac{(a-1)(a+3)}{2(a+3)} = \frac{a-1}{2} = \frac{1}{2}(a-1) = \frac{1}{2}a - \frac{1}{2}$

21 **a** $T = \frac{5a + 10b}{5} = \frac{5(a+2b)}{5} = a + 2b$

b $y = \frac{x^2 - 3x}{10x} = \frac{x(x-3)}{10x} = \frac{x-3}{10} = \frac{1}{10}(x-3) = \frac{1}{10}x - \frac{3}{10}$

c $A = \frac{p^2 + 4p}{p^2 + 5p + 4} = \frac{p(p+4)}{(p+1)(p+4)} = \frac{p}{p+1}$

22 **a** $N = \frac{3t^2 + 6t}{6t} = \frac{3t(t+2)}{6t} = \frac{t+2}{2} = \frac{1}{2}(t+2) = \frac{1}{2}t + 1$

b $y = \frac{x^2 - 5x + 4}{x^2 - 6x + 5} = \frac{(x-1)(x-4)}{(x-1)(x-5)} = \frac{x-4}{x-5}$

c $p = \frac{q^2 - 4}{q^2 + 4q + 4} = \frac{(q+2)(q-2)}{(q+2)(q+2)} = \frac{q-2}{q+2}$

23 In 3Ha is $\frac{4}{7} \cdot \frac{3}{4} = \frac{12}{28} = \frac{3}{7}$ deel van de leerlingen een meisje met een bijbaantje.
Dat zijn 12 leerlingen.
Dus in 3Ha zitten $\frac{12}{\frac{3}{7}} = 12 \cdot \frac{7}{3} = \frac{84}{3} = 28$ leerlingen.

Bladzijde 175

24 **a** $\frac{12a}{5c} : \frac{2}{3c} = \frac{12a}{5c} \cdot \frac{3c}{2} = \frac{36ac}{10c} = \frac{18a}{5} = 3\frac{3}{5}a$

b $\frac{\left(\frac{4p}{3}\right)}{2} = \frac{1}{2} \cdot \frac{4p}{3} = \frac{4p}{6} = \frac{2p}{3} = \frac{2}{3}p$

c $\frac{6}{\left(\frac{3}{2q}\right)} = 6 \cdot \frac{2q}{3} = \frac{12q}{3} = 4q$

25 **a** $\frac{3}{2a} \cdot \frac{6}{a} = \frac{18}{2a^2} = \frac{9}{a^2}$ **b** $-\frac{3}{x} \cdot \frac{-x}{3y} = \frac{3x}{3xy} = \frac{1}{y}$ **c** $\frac{8}{3a} \cdot \frac{5}{2a} = \frac{40}{6a^2} = \frac{20}{3a^2}$

26 **a** $\frac{3}{2a} : \frac{6}{a} = \frac{3}{2a} \cdot \frac{a}{6} = \frac{3a}{12a} = \frac{1}{4}$ **b** $\frac{1}{2a} : \frac{6}{c} = \frac{1}{2a} \cdot \frac{c}{6} = \frac{c}{12a}$ **c** $\frac{a}{9} : (3a) = \frac{a}{9} \cdot \frac{1}{3a} = \frac{a}{27a} = \frac{1}{27}$

Bladzijde 176

27 **a** $\frac{\left(\frac{3}{x}\right)}{8} = \frac{1}{8} \cdot \frac{3}{x} = \frac{3}{8x}$ **b** $\frac{9}{\left(\frac{3}{x}\right)} = 9 \cdot \frac{x}{3} = \frac{9x}{3} = 3x$ **c** $\frac{\left(\frac{2x}{5}\right)}{3} = \frac{1}{3} \cdot \frac{2x}{5} = \frac{2x}{15} = \frac{2}{15}x$

28 **a** $\frac{3}{2a} : \frac{3b}{a} = \frac{3}{2a} \cdot \frac{a}{3b} = \frac{3a}{6ab} = \frac{1}{2b}$ **c** $9 : \frac{3}{5a} = 9 \cdot \frac{5a}{3} = \frac{45a}{3} = 15a$ **e** $-6a : -\frac{2a}{3} = -6a \cdot -\frac{3}{2a} = \frac{18a}{2a} = 9$

b $\frac{\left(\frac{3}{2a}\right)}{4b} = \frac{1}{4b} \cdot \frac{3}{2a} = \frac{3}{8ab}$ **d** $\frac{2a}{3} \cdot 6a = \frac{12a^2}{3} = 4a^2$ **f** $\frac{9}{\left(\frac{6a}{5}\right)} = 9 \cdot \frac{5}{6a} = \frac{45}{6a} = \frac{15}{2a}$

29 $\frac{1}{6} + \frac{4}{5} = \frac{5}{30} + \frac{24}{30} = \frac{29}{30}$
Ze loopt $1 - \frac{29}{30} = \frac{1}{30}$ deel van de reisafstand.

Bladzijde 177

30 **a** $\frac{7p}{3q} - \frac{p}{3q} = \frac{6p}{3q} = \frac{2p}{q}$ **c** $\frac{2}{3x} + \frac{5}{2x} = \frac{4}{6x} + \frac{15}{6x} = \frac{19}{6x}$ **e** $\frac{4}{x} - 3 = \frac{4}{x} - \frac{3}{1} = \frac{4}{x} - \frac{3x}{x} = \frac{4 - 3x}{x}$

b $\frac{3}{5a} - \frac{2}{a} = \frac{3}{5a} - \frac{10}{5a} = -\frac{7}{5a}$ **d** $\frac{2}{a} + \frac{3}{a^2} = \frac{2a}{a^2} + \frac{3}{a^2} = \frac{2a + 3}{a^2}$ **f** $\frac{8}{3a} - \frac{3}{2b} = \frac{16b}{6ab} - \frac{9a}{6ab} = \frac{16b - 9a}{6ab}$

31 **a** $\frac{2}{x} + \frac{3}{y} \cdot \frac{2y}{7} = \frac{2}{x} + \frac{6y}{7y} = \frac{2}{x} + \frac{6}{7} = \frac{14}{7x} + \frac{6x}{7x} = \frac{14 + 6x}{7x}$

b $\frac{3}{a} : \frac{6}{b} - \frac{1}{4a} = \frac{3}{a} \cdot \frac{b}{6} - \frac{1}{4a} = \frac{3b}{6a} - \frac{1}{4a} = \frac{b}{2a} - \frac{1}{4a} = \frac{2b}{4a} - \frac{1}{4a} = \frac{2b - 1}{4a}$

c $2p \cdot \frac{3}{10pq} - 1 = \frac{6p}{10pq} - 1 = \frac{3}{5q} - \frac{5q}{5q} = \frac{3 - 5q}{5q}$

32 **a** Van type A wordt $\frac{12000}{60} = 200$ strekkende meter ingekocht.
Van type B wordt $\frac{12000}{40} = 300$ strekkende meter ingekocht.

b De gemiddelde prijs is $\frac{12000 + 12000}{200 + 300} = \frac{24000}{500} = 48$ euro per strekkende meter.

c Van type A wordt $\frac{b}{60}$ strekkende meter ingekocht.

Van type B wordt $\frac{b}{40}$ strekkende meter ingekocht.

Dus in totaal wordt er $\frac{b}{60} + \frac{b}{40}$ strekkende meter ingekocht.

Dus $l = \frac{b}{60} + \frac{b}{40} = \frac{2b}{120} + \frac{3b}{120} = \frac{5b}{120} = \frac{b}{24}$.

d De totale inkoopprijs is $b + b = 2b$ euro.

Het totale aantal ingekochte strekkende meter is l.

De gemiddelde prijs per strekkende meter is $\frac{\text{totale inkoopprijs}}{\text{totale aantal ingekochte strekkende meter}}$.

Dus gemiddeld $\frac{2b}{l} = \frac{2b}{\left(\frac{b}{24}\right)} = 2b \cdot \frac{24}{b} = \frac{48b}{b} = 48$ euro per strekkende meter.

33 **a** $\frac{2a}{3} + \frac{a-7}{4} = \frac{8a}{12} + \frac{3(a-7)}{12} = \frac{8a+3a-21}{12} = \frac{11a-21}{12}$

b $\frac{1}{2}a - \frac{2a+1}{3} = \frac{a}{2} - \frac{2a+1}{3} = \frac{3a}{6} - \frac{2(2a+1)}{6} = \frac{3a-2(2a+1)}{6} = \frac{3a-4a-2}{6} = \frac{-a-2}{6}$

c $5 - \frac{3x-y}{y} = \frac{5}{1} - \frac{3x-y}{y} = \frac{5y}{y} - \frac{3x-y}{y} = \frac{5y-(3x-y)}{y} = \frac{5y-3x+y}{y} = \frac{6y-3x}{y}$

d $\frac{2a-3}{a} + 2\frac{2}{3} = \frac{2a-3}{a} + \frac{8}{3} = \frac{3(2a-3)}{3a} + \frac{8a}{3a} = \frac{6a-9+8a}{3a} = \frac{14a-9}{3a}$

e $\frac{b+3}{b} - \frac{a-1}{a} = \frac{a(b+3)}{ab} - \frac{b(a-1)}{ab} = \frac{a(b+3)-b(a-1)}{ab} = \frac{ab+3a-ab+b}{ab} = \frac{3a+b}{ab}$

f $\frac{x+y}{x} - \frac{x-y}{y} = \frac{y(x+y)}{xy} - \frac{x(x-y)}{xy} = \frac{y(x+y)-x(x-y)}{xy} = \frac{xy+y^2-x^2+xy}{xy} = \frac{2xy+y^2-x^2}{xy}$

5.3 Machten herleiden

Bladzijde 178

34 Bewering II is waar.

35 **a** $6a^4 + 2a^5 - 7a^4 = -a^4 + 2a^5$

b $-3b^7c^2 \cdot 5bc^3 = -15b^8c^5$

c $-3p^5 \cdot -p^3 - 5p \cdot -2p^7 = 3p^8 + 10p^8 = 13p^8$

d $5x^3 \cdot -2y^5 - 3x^4 \cdot 2y^4 = -10x^3y^5 - 6x^4y^4$

36 **a** $5a^7 \cdot 7a^6 = 35a^{13}$

b $2a^5b^2 \cdot 3a^2b = 6a^7b^3$

c $x^5y \cdot -6x^7y^3 = -6x^{12}y^4$

d $-3a^9b^2 \cdot -5ab = 15a^{10}b^3$

e $-8a^6x^3 \cdot -5a^3x^{11} = 40a^9x^{14}$

f $9m^7n^3 \cdot -2m^2n^5 = -18m^9n^8$

Bladzijde 179

37 **a** $7x^8 + 3x^8 = 10x^8$

b $5a^4 - 3b^4 + 2a^4 = 7a^4 - 3b^4$

c $4a^2 \cdot 3a^5 + 2a^4 \cdot a^3 = 12a^7 + 2a^7 = 14a^7$

d $5x^4 \cdot 2x^6 - x^5 \cdot 2x^4 = 10x^{10} - 2x^9$

e $2x^6 \cdot 3x^6 - 5x^9 \cdot 4x^3 = 6x^{12} - 20x^{12} = -14x^{12}$

f $p^2 \cdot -3p^4 - 2p \cdot -4p^5 = -3p^6 + 8p^6 = 5p^6$

38 **a** $5x^3 \cdot 2x^5 - 3x^4 \cdot 2x^4 = 10x^8 - 6x^8 = 4x^8$

b $-3a^5b^3 \cdot -5a^2b^4 = 15a^7b^7$

c $m^4 \cdot 2m^3 + 4m^2 \cdot 3m^6 = 2m^7 + 12m^8$

d $4x^3 - 9y^2 + 5x^3 = 9x^3 - 9y^2$

e $4x^3 \cdot -9y^2 \cdot 5x^3 = -180x^6y^2$

f $4x^3 \cdot 9y^2 + x^3 \cdot -2y^2 = 36x^3y^2 - 2x^3y^2 = 34x^3y^2$

39 **a** $2a(3a^3 - 4a^5) = 6a^4 - 8a^6$

b $5x^3(x^4 - 2x) = 5x^7 - 10x^4$

c $a^3(a^2 + 2a) - a^4 = a^5 + 2a^4 - a^4 = a^5 + a^4$

d $3ab(2a - 3b) + ab^2 = 6a^2b - 9ab^2 + ab^2 = 6a^2b - 8ab^2$

e $p^3(2p^2 - 1) - p(p^4 + p) = 2p^5 - p^3 - p^5 - p^2 = p^5 - p^3 - p^2$

f $(a^4 + 5)^2 = a^8 + 10a^4 + 25$

g $(4x^3 + 9y)^2 - x^3 \cdot 2y^2 = 16x^6 + 72x^3y + 81y^2 - 2x^3y^2$

h $(4x^3 - 9y^2)(x^3 - 2y^2) = 4x^6 - 8x^3y^2 - 9x^3y^2 + 18y^4 = 4x^6 - 17x^3y^2 + 18y^4$

i $4x^3 \cdot 9y^2 - (x^3 - 2y)^2 = 36x^3y^2 - (x^6 - 4x^3y + 4y^2) = 36x^3y^2 - x^6 + 4x^3y - 4y^2$

Bladzijde 180

40 **a** $(5p^3)^2 = 25p^6$
b $-2(ab)^4 = -2a^4b^4$
c $(-3y^8)^4 = 81y^{32}$
d $(a^3)^4 \cdot 5a^2 = a^{12} \cdot 5a^2 = 5a^{14}$
e $3(x^2)^8 - 4(x^4)^4 = 3x^{16} - 4x^{16} = -x^{16}$
f $(2m^2)^3 \cdot 5(m^3)^3 = 8m^6 \cdot 5m^9 = 40m^{15}$

41 **a** $2(a^3)^2 + 3a^6 = 2a^6 + 3a^6 = 5a^6$
b $(3a^3)^3 - 5a^9 = 27a^9 - 5a^9 = 22a^9$
c $(a^4)^3 - 2(a^2)^6 = a^{12} - 2a^{12} = -a^{12}$
d $(5x^3)^2 \cdot 2x^4 = 25x^6 \cdot 2x^4 = 50x^{10}$
e $(-3p)^2 + 5p^2 = 9p^2 + 5p^2 = 14p^2$
f $2(a^3)^4 + (2a^4)^3 = 2a^{12} + 8a^{12} = 10a^{12}$

42 $a^4 \cdot a^3 = a^7$, dus $\frac{a^7}{a^3} = a^4$.

Bladzijde 181

43 **a** $\frac{x^{10}}{x^2} = x^8$
b $\frac{x^8}{x^8} = 1$
c $\frac{x^7}{x^6} = x$
d $\frac{5x^{10}}{x^2} = 5x^8$
e $\frac{3x^7}{3x^3} = x^4$
f $\frac{12x^7}{6x^7} = 2$
g $\frac{18x^3y^2}{-6xy^2} = -3x^2$
h $\frac{27x^3y^6}{9y^5} = 3x^3y$

44 **a** $2a^3 \cdot 8a^6 = 16a^9$
b $\frac{8a^6}{2a^3} = 4a^3$
c $2a^3 + 8a^6$ kan niet
d $(2a^3)^4 + 5(a^6)^2 = 16a^{12} + 5a^{12} = 21a^{12}$
e $\frac{2(a^3)^4}{8a^2} = \frac{2a^{12}}{8a^2} = \frac{a^{10}}{4} = \frac{1}{4}a^{10}$
f $(5x^2)^4 - 125x^8 = 625x^8 - 125x^8 = 500x^8$
g $\frac{21x^3 + 9x^3}{10x} = \frac{30x^3}{10x} = 3x^2$
h $16a^4 \cdot (-3a^3)^2 = 16a^4 \cdot 9a^6 = 144a^{10}$
i $3a \cdot a^7 - (a^4)^2 = 3a^8 - a^8 = 2a^8$
j $\frac{40x^6y^8}{8x^6y^7} = 5y$

45 **a** $\frac{(x^4)^2 + 5(x^2)^4}{3x^5} = \frac{x^8 + 5x^8}{3x^5} = \frac{6x^8}{3x^5} = 2x^3$
b $3(x^3)^4 + (x^6)^2 - 2x \cdot x^{11} = 3x^{12} + x^{12} - 2x^{12} = 2x^{12}$
c $-5(x^4)^2 \cdot (-3x)^2 \cdot x^6 = -5x^8 \cdot 9x^2 \cdot x^6 = -45x^{16}$
d $\frac{(5x)^2 - x^2}{-6x} = \frac{25x^2 - x^2}{-6x} = \frac{24x^2}{-6x} = -4x$
e $(5a^3)^2 - (-3a^2)^3 + a^6 = 25a^6 - -27a^6 + a^6 = 25a^6 + 27a^6 + a^6 = 53a^6$
f $(6x^5)^2 \cdot 6x^3 + (-4x^3)^3 \cdot x^4 = 36x^{10} \cdot 6x^3 + -64x^9 \cdot x^4 = 216x^{13} - 64x^{13} = 152x^{13}$

46 **a** $x^3(x^2 + x^3) - x^2(x^3 - 1) = x^5 + x^6 - x^5 + x^2 = x^6 + x^2$
b $3x^2(5x - 7) + 5x^2(x - 1) = 15x^3 - 21x^2 + 5x^3 - 5x^2 = 20x^3 - 26x^2$
c $(2x^3 - 3)^2 = 4x^6 - 12x^3 + 9$
d $(5x^4 - 1)^2 - x^6(x^2 - 1) = 25x^8 - 10x^4 + 1 - x^8 + x^6 = 24x^8 - 10x^4 + 1 + x^6$
e $(x^3 + x^2)(3x + x^6) = 3x^4 + x^9 + 3x^3 + x^8$
f $5x^4 - 3x^2(x^2 - 1) = 5x^4 - 3x^4 + 3x^2 = 2x^4 + 3x^2$

5.4 Wortels herleiden

Bladzijde 182

47 $\sqrt{4} \cdot \sqrt{25} = 2 \cdot 5 = 10$
$\sqrt{4 \cdot 25} = \sqrt{100} = 10$
Het valt op dat de uitkomsten gelijk zijn.

48
a $\sqrt{12} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$
b $\sqrt{75} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$
c $\sqrt{180} = \sqrt{36} \cdot \sqrt{5} = 6\sqrt{5}$
d $3\sqrt{200} = 3 \cdot \sqrt{100} \cdot \sqrt{2} = 3 \cdot 10\sqrt{2} = 30\sqrt{2}$
e $2\sqrt{45} = 2 \cdot \sqrt{9} \cdot \sqrt{5} = 2 \cdot 3\sqrt{5} = 6\sqrt{5}$
f $-3\sqrt{288} = -3 \cdot \sqrt{144} \cdot \sqrt{2} = -3 \cdot 12\sqrt{2} = -36\sqrt{2}$

49
a $5\sqrt{3} \cdot 3\sqrt{21} = 15\sqrt{63} = 15 \cdot \sqrt{9} \cdot \sqrt{7} = 15 \cdot 3\sqrt{7} = 45\sqrt{7}$
b $5\sqrt{6} \cdot 3\sqrt{2} = 15\sqrt{12} = 15 \cdot \sqrt{4} \cdot \sqrt{3} = 15 \cdot 2\sqrt{3} = 30\sqrt{3}$
c $5\sqrt{12} \cdot -3\sqrt{3} = -15\sqrt{36} = -15 \cdot 6 = -90$

Bladzijde 183

50
a $8\sqrt{5} \cdot -\sqrt{15} = -8\sqrt{75} = -8 \cdot \sqrt{25} \cdot \sqrt{3} = -8 \cdot 5\sqrt{3} = -40\sqrt{3}$
b $-3\sqrt{2} \cdot \frac{1}{2}\sqrt{32} = -1\frac{1}{2}\sqrt{64} = -1\frac{1}{2} \cdot 8 = -12$
c $6\sqrt{3} \cdot \sqrt{2} \cdot \frac{1}{2}\sqrt{12} = 3\sqrt{72} = 3 \cdot \sqrt{36} \cdot \sqrt{2} = 3 \cdot 6\sqrt{2} = 18\sqrt{2}$

51
a $\frac{6}{2} = 3$, want $3 \cdot 2 = 6$.
b $\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$, want $\sqrt{3} \cdot \sqrt{2} = \sqrt{6}$.

52
a $\frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$
b $\frac{12\sqrt{27}}{6\sqrt{3}} = 2\sqrt{9} = 2 \cdot 3 = 6$
c $\frac{2\sqrt{15}}{\sqrt{3}} = 2\sqrt{5}$
d $\frac{15\sqrt{7}}{25\sqrt{7}} = \frac{15}{25} = \frac{3}{5}$
e $\frac{24\sqrt{18}}{18\sqrt{6}} = \frac{4}{3}\sqrt{3} = 1\frac{1}{3}\sqrt{3}$
f $\frac{\sqrt{24}}{3\sqrt{3}} = \frac{1}{3}\sqrt{8} = \frac{1}{3} \cdot \sqrt{4} \cdot \sqrt{2} = \frac{1}{3} \cdot 2\sqrt{2} = \frac{2}{3}\sqrt{2}$

Bladzijde 184

53
a $\sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{\sqrt{16}} = \frac{\sqrt{3}}{4} = \frac{1}{4}\sqrt{3}$
b $\sqrt{\frac{45}{81}} = \frac{\sqrt{45}}{\sqrt{81}} = \frac{\sqrt{9} \cdot \sqrt{5}}{9} = \frac{3\sqrt{5}}{9} = \frac{1}{3}\sqrt{5}$
c $\sqrt{1\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{\sqrt{4}} = \frac{\sqrt{5}}{2} = \frac{1}{2}\sqrt{5}$
d $\sqrt{3\frac{24}{25}} = \sqrt{\frac{99}{25}} = \frac{\sqrt{99}}{\sqrt{25}} = \frac{\sqrt{9} \cdot \sqrt{11}}{5} = \frac{3\sqrt{11}}{5} = \frac{3}{5}\sqrt{11}$

54
a $\frac{15\sqrt{3}}{25} = \frac{3}{5}\sqrt{3}$
b $\sqrt{1\frac{11}{25}} = \sqrt{\frac{36}{25}} = \frac{\sqrt{36}}{\sqrt{25}} = \frac{6}{5} = 1\frac{1}{5}$
c $\frac{3\sqrt{12}}{12\sqrt{3}} = \frac{1}{4}\sqrt{4} = \frac{1}{4} \cdot 2 = \frac{1}{2}$
d $\sqrt{3\frac{1}{9}} = \sqrt{\frac{28}{9}} = \frac{\sqrt{28}}{\sqrt{9}} = \frac{\sqrt{4} \cdot \sqrt{7}}{3} = \frac{2\sqrt{7}}{3} = \frac{2}{3}\sqrt{7}$
e $\frac{2\sqrt{50}}{3} = \frac{2}{3} \cdot \sqrt{25} \cdot \sqrt{2} = \frac{2}{3} \cdot 5\sqrt{2} = \frac{10}{3}\sqrt{2} = 3\frac{1}{3}\sqrt{2}$
f $\sqrt{1\frac{21}{100}} = \sqrt{\frac{121}{100}} = \frac{\sqrt{121}}{\sqrt{100}} = \frac{11}{10} = 1\frac{1}{10}$
g $\frac{\sqrt{72}}{6\sqrt{6}} = \frac{1}{6}\sqrt{12} = \frac{1}{6} \cdot \sqrt{4} \cdot \sqrt{3} = \frac{1}{6} \cdot 2\sqrt{3} = \frac{1}{3}\sqrt{3}$
h $\sqrt{2\frac{1}{16}} = \sqrt{\frac{33}{16}} = \frac{\sqrt{33}}{\sqrt{16}} = \frac{\sqrt{33}}{4} = \frac{1}{4}\sqrt{33}$

55 $\sqrt{18} + \sqrt{2} = \sqrt{9} \cdot \sqrt{2} + \sqrt{2} = 3\sqrt{2} + \sqrt{2} = 4\sqrt{2}$

Bladzijde 185

56 **a** $7\sqrt{2} - \sqrt{8} = 7\sqrt{2} - \sqrt{4} \cdot \sqrt{2} = 7\sqrt{2} - 2\sqrt{2} = 5\sqrt{2}$

b $\sqrt{12} - \sqrt{3} = \sqrt{4} \cdot \sqrt{3} - \sqrt{3} = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

c $2\sqrt{90} - 3\sqrt{40} = 2 \cdot \sqrt{9} \cdot \sqrt{10} - 3 \cdot \sqrt{4} \cdot \sqrt{10} = 2 \cdot 3\sqrt{10} - 3 \cdot 2\sqrt{10} = 6\sqrt{10} - 6\sqrt{10} = 0$

57 **a** $3\sqrt{60} + 2\sqrt{135} = 3 \cdot \sqrt{4} \cdot \sqrt{15} + 2 \cdot \sqrt{9} \cdot \sqrt{15} = 3 \cdot 2\sqrt{15} + 2 \cdot 3\sqrt{15} = 6\sqrt{15} + 6\sqrt{15} = 12\sqrt{15}$

b $8\sqrt{3} \cdot \sqrt{21} - 2\sqrt{7} = 8\sqrt{63} - 2\sqrt{7} = 8 \cdot \sqrt{9} \cdot \sqrt{7} - 2\sqrt{7} = 8 \cdot 3\sqrt{7} - 2\sqrt{7} = 24\sqrt{7} - 2\sqrt{7} = 22\sqrt{7}$

c $\sqrt{2} \cdot \frac{1}{2}\sqrt{32} - \sqrt{150} = \frac{1}{2}\sqrt{64} - \sqrt{25} \cdot \sqrt{6} = \frac{1}{2} \cdot 8 - 5\sqrt{6} = 4 - 5\sqrt{6}$

d $\sqrt{54} - 2\sqrt{3} \cdot \sqrt{2} + \frac{1}{2}\sqrt{6} = \sqrt{9} \cdot \sqrt{6} - 2\sqrt{6} + \frac{1}{2}\sqrt{6} = 3\sqrt{6} - 2\sqrt{6} + \frac{1}{2}\sqrt{6} = 1\frac{1}{2}\sqrt{6}$

e $\frac{20\sqrt{7}}{2\sqrt{7}} - \sqrt{18} \cdot \frac{1}{2}\sqrt{2} = 10 - \frac{1}{2}\sqrt{36} = 10 - \frac{1}{2} \cdot 6 = 10 - 3 = 7$

f $\frac{6\sqrt{40}}{3\sqrt{2}} - 5\sqrt{125} = 2\sqrt{20} - 5 \cdot \sqrt{25} \cdot \sqrt{5} = 2 \cdot \sqrt{4} \cdot \sqrt{5} - 5 \cdot 5\sqrt{5} = 2 \cdot 2\sqrt{5} - 25\sqrt{5} =$
$4\sqrt{5} - 25\sqrt{5} = -21\sqrt{5}$

58 **a** $\frac{10}{\sqrt{5}} = \frac{10 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$

b $\sqrt{4\frac{1}{6}} = \sqrt{\frac{25}{6}} = \frac{\sqrt{25}}{\sqrt{6}} = \frac{5 \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{5\sqrt{6}}{6} = \frac{5}{6}\sqrt{6}$

c $\frac{21}{3\sqrt{7}} = \frac{7}{\sqrt{7}} = \frac{7 \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}} = \frac{7\sqrt{7}}{7} = \sqrt{7}$

d $4\sqrt{\frac{1}{2}} = 4 \cdot \frac{\sqrt{1}}{\sqrt{2}} = 4 \cdot \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = 4 \cdot \frac{\sqrt{2}}{2} = 4 \cdot \frac{1}{2}\sqrt{2} = 2\sqrt{2}$

e $\frac{9}{2\sqrt{6}} = \frac{9 \cdot \sqrt{6}}{2\sqrt{6} \cdot \sqrt{6}} = \frac{9\sqrt{6}}{2 \cdot 6} = \frac{9\sqrt{6}}{12} = \frac{3}{4}\sqrt{6}$

f $\sqrt{3\frac{1}{3}} = \sqrt{\frac{10}{3}} = \frac{\sqrt{10}}{\sqrt{3}} = \frac{\sqrt{10} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{30}}{3} = \frac{1}{3}\sqrt{30}$

59 **a** $\frac{10}{\sqrt{2}} + 3\sqrt{8} = \frac{10 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} + 3 \cdot \sqrt{4} \cdot \sqrt{2} = \frac{10\sqrt{2}}{2} + 3 \cdot 2\sqrt{2} = 5\sqrt{2} + 6\sqrt{2} = 11\sqrt{2}$

b $\frac{\sqrt{8} + \sqrt{32}}{12} = \frac{\sqrt{4} \cdot \sqrt{2} + \sqrt{16} \cdot \sqrt{2}}{12} = \frac{2\sqrt{2} + 4\sqrt{2}}{12} = \frac{6\sqrt{2}}{12} = \frac{1}{2}\sqrt{2}$

c $\sqrt{12\frac{1}{2}} - \sqrt{4\frac{1}{2}} = \sqrt{\frac{25}{2}} - \sqrt{\frac{9}{2}} = \frac{\sqrt{25}}{\sqrt{2}} - \frac{\sqrt{9}}{\sqrt{2}} = \frac{5}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

d $\frac{2}{5}\sqrt{15} \cdot \frac{1}{4}\sqrt{5} = \frac{2}{20}\sqrt{75} = \frac{1}{10} \cdot \sqrt{25} \cdot \sqrt{3} = \frac{1}{10} \cdot 5\sqrt{3} = \frac{1}{2}\sqrt{3}$

e $\sqrt{24} - \frac{12}{\sqrt{6}} = \sqrt{4} \cdot \sqrt{6} - \frac{12 \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = 2\sqrt{6} - \frac{12\sqrt{6}}{6} = 2\sqrt{6} - 2\sqrt{6} = 0$

f $\sqrt{\frac{5}{12}} \cdot \sqrt{\frac{1}{15}} = \sqrt{\frac{5}{180}} = \sqrt{\frac{1}{36}} = \frac{\sqrt{1}}{\sqrt{36}} = \frac{1}{6}$

5.5 Vergelijkingen met twee variabelen

Bladzijde 186

60 **a** 3 zakken bruine bollen kosten $3 \cdot 4 = 12$ euro.
6 zakken krentenbollen kosten $24 - 12 = 12$ euro.
Een zak krentenbollen kost dan $12 : 6 = 2$ euro.

b 3 zakken bruine bollen kosten $3 \cdot 2 = 6$ euro.
6 zakken krentenbollen kosten $24 - 6 = 18$ euro.
Een zak krentenbollen kost dan $18 : 6 = 3$ euro.

c Drie zakken bruine bollen kosten dan $3y$ euro.
Samen kosten ze 24 euro, dus $6x + 3y = 24$.

61 **a** $2x + y = 7$
$y = -2x + 7$

b $2x - y = -7$
$-y = -2x - 7$
$y = 2x + 7$

c $3x + 2y = 8$
$2y = -3x + 8$
$y = -1\frac{1}{2}x + 4$

Bladzijde 187

62 **a** $3x - 4y = 12$
$-4y = -3x + 12$
$y = \frac{3}{4}x - 3$

b $5x + 2y = 14$
$2y = -5x + 14$
$y = -2\frac{1}{2}x + 7$

c $6x - 4y = -12$
$-4y = -6x - 12$
$y = 1\frac{1}{2}x + 3$

63 **a** $3x + 2y = 6$
$2y = -3x + 6$
$y = -1\frac{1}{2}x + 3$

b $\text{rc}_l = -1\frac{1}{2}$

64 **a** m: $5x - y = 10$
m: $-y = -5x + 10$
m: $y = 5x - 10$
$\text{rc}_m = 5$

n: $x + 2y = 13$
n: $2y = -x + 13$
n: $y = -\frac{1}{2}x + 6\frac{1}{2}$
$\text{rc}_n = -\frac{1}{2}$

b $5x - 10 = -\frac{1}{2}x + 6\frac{1}{2}$
$5x + \frac{1}{2}x = 6\frac{1}{2} + 10$
$5\frac{1}{2}x = 16\frac{1}{2}$
$x = 3$
$x = 3$ geeft $y = 5 \cdot 3 - 10 = 15 - 10 = 5$
Dus $S(3, 5)$.

65 **a** $6x + 4y = 10{,}4$

b $x + 2y = 85$

66 **a** $1{,}5 \cdot 8 + 2y = 28$
$12 + 2y = 28$
$2y = 28 - 12$
$2y = 16$
$y = 8$
Je weet nu dat Hans acht flessen cola koopt.

b $x = 12$ geeft $1{,}5 \cdot 12 + 2y = 28$
$18 + 2y = 28$
$2y = 28 - 18$
$2y = 10$
$y = 5$
In dat geval koopt hij vijf flessen cola.

c $y = 2$ geeft $1{,}5x + 2 \cdot 2 = 28$
$1{,}5x + 4 = 28$
$1{,}5x = 28 - 4$
$1{,}5x = 24$
$x = 16$
In dat geval koopt hij zestien pakken melk.

67 **a** $12x + 16y = 5740$

b $x = 225$ geeft $225 \cdot 12 + 16y = 5740$

$2700 + 16y = 5740$

$16y = 5740 - 2700$

$16y = 3040$

$y = 190$

Dus er zijn dan 190 kaarten van 16 euro verkocht.

68 **a** $7{,}5x + 10y = 1300$

b $x + y = 150$

c $7{,}5x + 10y = 1300$ $\qquad$ $x + y = 150$

$10y = -7{,}5x + 1300$ $\qquad$ $y = -x + 150$

$y = -0{,}75x + 130$

$-0{,}75x + 130 = -x + 150$

$-0{,}75x + x = 150 - 130$

$0{,}25x = 20$

$x = 80$

$x = 80$ geeft $y = -80 + 150 = 70$

Er zijn die avond 80 jongeren en 70 volwassenen.

Gemengde opgaven

Bladzijde 188

1 **a** $4(2n-1)^2 - (4n)^2 = 4(4n^2 - 4n + 1) - 16n^2 = 16n^2 - 16n + 4 - 16n^2 = -16n + 4$

b $2(1-q)(q+2) - 2(3-q) = 2(q + 2 - q^2 - 2q) - 6 + 2q = 2q + 4 - 2q^2 - 4q - 6 + 2q = -2 - 2q^2$

c $8(a-3b)(a+3b) - 4a \cdot 2a = 8(a^2 - 9b^2) - 8a^2 = 8a^2 - 72b^2 - 8a^2 = -72b^2$

d $3(5x-2y)^2 + 4y(15x - 3y) = 3(25x^2 - 20xy + 4y^2) + 60xy - 12y^2 =$
$75x^2 - 60xy + 12y^2 + 60xy - 12y^2 = 75x^2$

2 **a** $R = 4(a+1)(3a-4) - (2a+1)^2$

$R = 4(3a^2 - 4a + 3a - 4) - (4a^2 + 4a + 1)$

$R = 12a^2 - 16a + 12a - 16 - 4a^2 - 4a - 1$

$R = 8a^2 - 8a - 17$

b $y = \dfrac{-4x^2 - 6x}{10x + 15} = \dfrac{-2x(2x+3)}{5(2x+3)} = \dfrac{-2x}{5} = -\dfrac{2}{5}x$

c $W = -(1-5p)(3+2p) - 4(3-p)^2$

$W = -(3 + 2p - 15p - 10p^2) - 4(9 - 6p + p^2)$

$W = -3 - 2p + 15p + 10p^2 - 36 + 24p - 4p^2$

$W = -39 + 37p + 6p^2$

d $G = \dfrac{n^2 - 16}{n^2 - 7n + 12} = \dfrac{(n-4)(n+4)}{(n-3)(n-4)} = \dfrac{n+4}{n-3}$

3 **a** $\dfrac{3}{2a} : \dfrac{1}{6a} = \dfrac{3}{2a} \cdot \dfrac{6a}{1} = \dfrac{18a}{2a} = 9$

b $2\frac{3}{4} + \dfrac{4}{3a} = \dfrac{11}{4} + \dfrac{4}{3a} = \dfrac{33a}{12a} + \dfrac{16}{12a} = \dfrac{33a + 16}{12a}$

c $\dfrac{\left(\frac{4}{3x}\right)}{2x} = \dfrac{1}{2x} \cdot \dfrac{4}{3x} = \dfrac{4}{6x^2} = \dfrac{2}{3x^2}$

d $\dfrac{3x}{2y} - \dfrac{4}{y^2} = \dfrac{3xy}{2y^2} - \dfrac{8}{2y^2} = \dfrac{3xy - 8}{2y^2}$

e $3x \cdot \dfrac{a}{12x} - \dfrac{12a^6}{4a^5} = \dfrac{3ax}{12x} - \dfrac{12a^6}{4a^5} = \dfrac{a}{4} - 3a = \frac{1}{4}a - 3a = -2\frac{3}{4}a$

f $\dfrac{4p^2}{\left(\frac{16p}{3}\right)} - \dfrac{pq}{2q} = 4p^2 \cdot \dfrac{3}{16p} - \dfrac{p}{2} = \dfrac{12p^2}{16p} - \frac{1}{2}p = \dfrac{3p}{4} - \frac{1}{2}p = \frac{3}{4}p - \frac{2}{4}p = \frac{1}{4}p$

4 **a** $(-3x^2)^3 \cdot 2(x^5)^2 = -27x^6 \cdot 2x^{10} = -54x^{16}$

b $6(x^2)^9 - (2x^6)^3 = 6x^{18} - 8x^{18} = -2x^{18}$

c $\dfrac{6q^5 - 4q^5}{(2q)^2} = \dfrac{2q^5}{4q^2} = \dfrac{q^3}{2} = \frac{1}{2}q^3$

d $-8a^2x^5 \cdot -3ax^2 = 24a^3x^7$

e $3x^5 \cdot 2y^3 - xy^2 \cdot -4x^4y = 6x^5y^3 + 4x^5y^3 = 10x^5y^3$

f $\dfrac{6x^{24}y^5}{24x^6y^5} = \dfrac{x^{18}}{4} = \frac{1}{4}x^{18}$

5 **a** $\dfrac{\sqrt{84}}{3\sqrt{7}} = \frac{1}{3}\sqrt{12} = \frac{1}{3} \cdot \sqrt{4} \cdot \sqrt{3} = \frac{1}{3} \cdot 2\sqrt{3} = \frac{2}{3}\sqrt{3}$

b $\dfrac{6\sqrt{5} \cdot 3\sqrt{27}}{8\sqrt{3}} = \dfrac{18\sqrt{135}}{8\sqrt{3}} = \frac{18}{8}\sqrt{45} = \frac{9}{4} \cdot \sqrt{9} \cdot \sqrt{5} = 2\frac{1}{4} \cdot 3\sqrt{5} = 6\frac{3}{4}\sqrt{5}$

c $3\sqrt{2\frac{7}{9}} = 3\sqrt{\frac{25}{9}} = 3 \cdot \dfrac{\sqrt{25}}{\sqrt{9}} = 3 \cdot \frac{5}{3} = 5$

d $3\frac{1}{2}\sqrt{2\frac{1}{49}} = 3\frac{1}{2}\sqrt{\frac{99}{49}} = \frac{7}{2} \cdot \dfrac{\sqrt{99}}{\sqrt{49}} = \dfrac{7 \cdot \sqrt{9} \cdot \sqrt{11}}{2 \cdot 7} = \dfrac{3\sqrt{11}}{2} = \frac{3}{2}\sqrt{11} = 1\frac{1}{2}\sqrt{11}$

Bladzijde 189

6 **a** $2x + 7y = 15$

$2x = -7y + 15$

$x = -3\frac{1}{2}y + 7\frac{1}{2}$

b $\dfrac{x}{3} + \dfrac{y}{2} = 1$

$\frac{1}{3}x + \frac{1}{2}y = 1$

$\frac{1}{2}y = -\frac{1}{3}x + 1$

$y = -\frac{2}{3}x + 2$

7 m: $x + 2y = 8$ | n: $5x - 2y = 10$

m: $2y = -x + 8$ | n: $-2y = -5x + 10$

m: $y = -\frac{1}{2}x + 4$ | n: $y = 2\frac{1}{2}x - 5$

$-\frac{1}{2}x + 4 = 0$ | $2\frac{1}{2}x - 5 = 0$

$-\frac{1}{2}x = -4$ | $2\frac{1}{2}x = 5$

$x = 8$ | $x = 2$

$A(8, 0)$ | $B(2, 0)$

Dus $AB = 8 - 2 = 6$.

$2\frac{1}{2}x - 5 = -\frac{1}{2}x + 4$

$2\frac{1}{2}x + \frac{1}{2}x = 4 + 5$

$3x = 9$

$x = 3$

$x = 3$ geeft $y = 2\frac{1}{2} \cdot 3 - 5 = 7\frac{1}{2} - 5 = 2\frac{1}{2}$, dus $S(3, 2\frac{1}{2})$.

opp $\triangle ABS = \frac{1}{2} \cdot 6 \cdot 2\frac{1}{2} = 7\frac{1}{2}$

8 **a** $25x + 35y = 65\,500$

b $y = 1300$ geeft $25x + 35 \cdot 1300 = 65\,500$

$25x + 45\,500 = 65\,500$

$25x = 65\,500 - 45\,500$

$25x = 20\,000$

$x = 800$

Dus in totaal zijn er dan $1300 + 800 = 2100$ kaarten verkocht.

9 **a** $(-3a^3b)^3 \cdot \frac{1}{9(a^2b)^2} = -27a^9b^3 \cdot \frac{1}{9a^4b^2} = \frac{-27a^9b^3}{9a^4b^2} = -3a^5b$

b $(x^3 + 2x)^2 = x^6 + 4x^4 + 4x^2$

c $2p^6\left(\frac{1}{p^5} - \frac{1}{p}\right) = \frac{2p^6}{p^5} - \frac{2p^6}{p} = 2p - 2p^5$

d $\frac{5\sqrt{63}}{15\sqrt{21}} + \sqrt{\frac{3}{4}} = \frac{1}{3}\sqrt{3} + \frac{\sqrt{3}}{\sqrt{4}} = \frac{1}{3}\sqrt{3} + \frac{\sqrt{3}}{2} = \frac{1}{3}\sqrt{3} + \frac{1}{2}\sqrt{3} = \frac{2}{6}\sqrt{3} + \frac{3}{6}\sqrt{3} = \frac{5}{6}\sqrt{3}$

e $\frac{\sqrt{80}}{3\sqrt{8}} = \frac{1}{3}\sqrt{10}$

f $\frac{5}{\sqrt{3}} - 2\sqrt{27} = \frac{5 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} - 2 \cdot \sqrt{9} \cdot \sqrt{3} = \frac{5\sqrt{3}}{3} - 2 \cdot 3\sqrt{3} = \frac{5}{3}\sqrt{3} - 6\sqrt{3} = 1\frac{2}{3}\sqrt{3} - 6\sqrt{3} = -4\frac{1}{3}\sqrt{3}$

g $\sqrt{\frac{1}{2}} + \sqrt{18} = \frac{\sqrt{1}}{\sqrt{2}} + \sqrt{9} \cdot \sqrt{2} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} + 3\sqrt{2} = \frac{\sqrt{2}}{2} + 3\sqrt{2} = \frac{1}{2}\sqrt{2} + 3\sqrt{2} = 3\frac{1}{2}\sqrt{2}$

h $\frac{(-4a^4)^3 \cdot (b^3)^2}{2(a^2b^2)^2} = \frac{-64a^{12}b^6}{2a^4b^4} = -32a^8b^2$

i $(x^5 - x^4)\left(\frac{1}{x^3} + \frac{1}{x^2}\right) = \frac{x^5}{x^3} + \frac{x^5}{x^2} - \frac{x^4}{x^3} - \frac{x^4}{x^2} = x^2 + x^3 - x - x^2 = x^3 - x$

j $\frac{3}{x-1} - \frac{2}{x+2} = \frac{3(x+2)}{(x-1)(x+2)} - \frac{2(x-1)}{(x-1)(x+2)} = \frac{3(x+2) - 2(x-1)}{(x-1)(x+2)} = \frac{3x + 6 - 2x + 2}{(x-1)(x+2)} = \frac{x+8}{(x-1)(x+2)}$

k $6 - \frac{3-5x}{x} = \frac{6}{1} - \frac{3-5x}{x} = \frac{6x}{x} - \frac{3-5x}{x} = \frac{6x - (3-5x)}{x} = \frac{6x - 3 + 5x}{x} = \frac{11x - 3}{x}$

l $\frac{4x^2}{4x^3 + 10x} = \frac{4x^2}{2x(2x^2+5)} = \frac{2x}{2x^2+5}$

Diagnostische toets

Bladzijde 192

1 **a** $5a + 3(2a-1)(a+2) = 5a + 3(2a^2 + 4a - a - 2) = 5a + 6a^2 + 12a - 3a - 6 = 14a + 6a^2 - 6$

b $4(a-1)^2 - (2a)^2 = 4(a^2 - 2a + 1) - 4a^2 = 4a^2 - 8a + 4 - 4a^2 = -8a + 4$

c $2(5-x)(4x+3) - (2x)^2 = 2(20x + 15 - 4x^2 - 3x) - 4x^2 = 40x + 30 - 8x^2 - 6x - 4x^2 = 34x + 30 - 12x^2$

d $-(3p-2)(3p+2) = -(9p^2 - 4) = -9p^2 + 4$

2 **a** $(p+3q)^2 - (p-3q)^2 = p^2 + 6pq + 9q^2 - (p^2 - 6pq + 9q^2) = p^2 + 6pq + 9q^2 - p^2 + 6pq - 9q^2 = 12pq$

b $12(n-1)^2 - 3(2n+1)(2n-1) = 12(n^2 - 2n + 1) - 3(4n^2 - 1) = 12n^2 - 24n + 12 - 12n^2 + 3 = -24n + 15$

3 **a** $\frac{5a}{10} + \frac{12ac}{4c} = \frac{a}{2} + 3a = \frac{1}{2}a + 3a = 3\frac{1}{2}a$

b $\frac{4xyz}{2xz} - \frac{6yz}{10z} = 2y - \frac{3y}{5} = 2y - \frac{3}{5}y = 1\frac{2}{5}y$

4 **a** $N = \frac{10t}{30t^2 - 5t} = \frac{10t}{5t(6t-1)} = \frac{2}{6t-1}$

b $y = \frac{x^2 + 2x - 8}{x^2 - 3x + 2} = \frac{(x+4)(x-2)}{(x-1)(x-2)} = \frac{x+4}{x-1}$

5 **a** $\dfrac{\left(\frac{3}{a}\right)}{4} = \dfrac{1}{4} \cdot \dfrac{3}{a} = \dfrac{3}{4a}$

b $\dfrac{7}{10p} : \dfrac{2q}{5p} = \dfrac{7}{10p} \cdot \dfrac{5p}{2q} = \dfrac{35p}{20pq} = \dfrac{7}{4q}$

c $\dfrac{5}{3p} - \dfrac{7}{2q} = \dfrac{10q}{6pq} - \dfrac{21p}{6pq} = \dfrac{10q - 21p}{6pq}$

d $\dfrac{\left(\frac{5}{2c}\right)}{10} = \dfrac{1}{10} \cdot \dfrac{5}{2c} = \dfrac{5}{20c} = \dfrac{1}{4c}$

e $\dfrac{4}{\left(\frac{6}{5p}\right)} = 4 \cdot \dfrac{5p}{6} = \dfrac{20p}{6} = \dfrac{10p}{3} = \frac{10}{3}p = 3\frac{1}{3}p$

f $\dfrac{3}{2a} - \dfrac{2}{5} \cdot \dfrac{3}{4a} = \dfrac{3}{2a} - \dfrac{6}{20a} = \dfrac{30}{20a} - \dfrac{6}{20a} = \dfrac{24}{20a} = \dfrac{6}{5a}$

6 **a** $\dfrac{3 - 4x}{x} - 1\frac{1}{2} = \dfrac{3 - 4x}{x} - \dfrac{3}{2} = \dfrac{2(3 - 4x)}{2x} - \dfrac{3x}{2x} = \dfrac{2(3 - 4x) - 3x}{2x} = \dfrac{6 - 8x - 3x}{2x} = \dfrac{6 - 11x}{2x}$

b $\dfrac{p + 5}{p} - \dfrac{q - 3}{q} = \dfrac{q(p + 5)}{pq} - \dfrac{p(q - 3)}{pq} = \dfrac{q(p + 5) - p(q - 3)}{pq} = \dfrac{pq + 5q - pq + 3p}{pq} = \dfrac{5q + 3p}{pq}$

7 **a** $2a^2 \cdot 5a^5 + 3a^4 \cdot -4a^3 = 10a^7 - 12a^7 = -2a^7$

b $-6xy^5 \cdot -5x^3y^2 = 30x^4y^7$

c $\dfrac{24m^7n^4}{8m^2n^3} = 3m^5n$

d $3p^6 - 4q^6 + 7p^6 = 10p^6 - 4q^6$

e $3(n^2)^6 - 4(n^4)^3 = 3n^{12} - 4n^{12} = -n^{12}$

f $\dfrac{(3x^3)^2}{27x} = \dfrac{9x^6}{27x} = \dfrac{x^5}{3} = \frac{1}{3}x^5$

8 **a** $4x^2(x^4 - 3x) - (-2x^3)^2 = 4x^6 - 12x^3 - 4x^6 = -12x^3$

b $\dfrac{(2x^2)^5 - 2x^{10}}{3x^3} = \dfrac{32x^{10} - 2x^{10}}{3x^3} = \dfrac{30x^{10}}{3x^3} = 10x^7$

c $(2a^5 - 3b)^2 - 3a^2b \cdot -4a^3 = 4a^{10} - 12a^5b + 9b^2 + 12a^5b = 4a^{10} + 9b^2$

d $\dfrac{(6x^3)^2 - 9(x^2)^3}{6x^2} = \dfrac{36x^6 - 9x^6}{6x^2} = \dfrac{27x^6}{6x^2} = \dfrac{9x^4}{2} = \frac{9}{2}x^4 = 4\frac{1}{2}x^4$

Bladzijde 193

9 **a** $2\sqrt{10} \cdot -5\sqrt{6} = -10\sqrt{60} = -10 \cdot \sqrt{4} \cdot \sqrt{15} = -10 \cdot 2\sqrt{15} = -20\sqrt{15}$

b $\dfrac{\sqrt{20}}{4\sqrt{5}} = \frac{1}{4}\sqrt{4} = \frac{1}{4} \cdot 2 = \frac{1}{2}$

c $\sqrt{1\frac{7}{9}} = \sqrt{\frac{16}{9}} = \dfrac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3} = 1\frac{1}{3}$

d $\dfrac{5\sqrt{24}}{\sqrt{2}} = 5\sqrt{12} = 5 \cdot \sqrt{4} \cdot \sqrt{3} = 5 \cdot 2\sqrt{3} = 10\sqrt{3}$

e $\sqrt{1\frac{7}{25}} = \sqrt{\frac{32}{25}} = \dfrac{\sqrt{32}}{\sqrt{25}} = \dfrac{\sqrt{16} \cdot \sqrt{2}}{5} = \dfrac{4\sqrt{2}}{5} = \frac{4}{5}\sqrt{2}$

f $\dfrac{15\sqrt{108}}{24\sqrt{3}} = \frac{15}{24}\sqrt{36} = \frac{5}{8} \cdot 6 = \frac{30}{8} = \frac{15}{4} = 3\frac{3}{4}$

10 **a** $\sqrt{5} \cdot \sqrt{10} - 3\sqrt{8} = \sqrt{50} - 3 \cdot \sqrt{4} \cdot \sqrt{2} = \sqrt{25} \cdot \sqrt{2} - 3 \cdot 2\sqrt{2} = 5\sqrt{2} - 6\sqrt{2} = -\sqrt{2}$

b $\sqrt{90} - 4\sqrt{2} \cdot \sqrt{5} + \frac{1}{2}\sqrt{10} = \sqrt{9} \cdot \sqrt{10} - 4\sqrt{10} + \frac{1}{2}\sqrt{10} = 3\sqrt{10} - 4\sqrt{10} + \frac{1}{2}\sqrt{10} = -\frac{1}{2}\sqrt{10}$

c $\frac{3}{\sqrt{6}} - \sqrt{24} = \frac{3 \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} - \sqrt{4} \cdot \sqrt{6} = \frac{3\sqrt{6}}{6} - 2\sqrt{6} = \frac{3}{6}\sqrt{6} - 2\sqrt{6} = \frac{1}{2}\sqrt{6} - 2\sqrt{6} = -1\frac{1}{2}\sqrt{6}$

d $\sqrt{10\frac{2}{3}} - \sqrt{2\frac{2}{3}} = \sqrt{\frac{32}{3}} - \sqrt{\frac{8}{3}} = \frac{\sqrt{32}}{\sqrt{3}} - \frac{\sqrt{8}}{\sqrt{3}} = \frac{\sqrt{16} \cdot \sqrt{2}}{\sqrt{3}} - \frac{\sqrt{4} \cdot \sqrt{2}}{\sqrt{3}} = \frac{4\sqrt{2}}{\sqrt{3}} - \frac{2\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{3}} =$
$\frac{2\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{6}}{3} = \frac{2}{3}\sqrt{6}$

e $\sqrt{5} \cdot \sqrt{15} - \sqrt{\frac{1}{3}} = \sqrt{75} - \frac{\sqrt{1}}{\sqrt{3}} = \sqrt{25} \cdot \sqrt{3} - \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = 5\sqrt{3} - \frac{\sqrt{3}}{3} = 5\sqrt{3} - \frac{1}{3}\sqrt{3} = 4\frac{2}{3}\sqrt{3}$

f $\frac{\sqrt{27} - \sqrt{3}}{\sqrt{2}} = \frac{\sqrt{9} \cdot \sqrt{3} - \sqrt{3}}{\sqrt{2}} = \frac{3\sqrt{3} - \sqrt{3}}{\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{2}} = \frac{2\sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{2\sqrt{6}}{2} = \sqrt{6}$

11 **a**

$3x - y = 11$	$x - 2y = 12$
$-y = -3x + 11$	$-2y = -x + 12$
$y = 3x - 11$	$y = \frac{1}{2}x - 6$
Dus $rc_k = 3$.	Dus $rc_l = \frac{1}{2}$.

b $3x - 11 = \frac{1}{2}x - 6$
$3x - \frac{1}{2}x = -6 + 11$
$2\frac{1}{2}x = 5$
$x = 2$
$x = 2$ geeft $y = 3 \cdot 2 - 11 = 6 - 11 = -5$
Dus $S(2, -5)$.

12 **a** $2{,}5x + 2y = 125$

b $x = 18$ geeft $2{,}5 \cdot 18 + 2y = 125$
$45 + 2y = 125$
$2y = 125 - 45$
$2y = 80$
$y = 40$
Peter heeft 40 bakjes frambozen verkocht.

Herhaling

Bladzijde 194

1 **a** $3(x + 1)(x - 3) = 3(x^2 - 3x + x - 3) = 3(x^2 - 2x - 3) = 3x^2 - 6x - 9$

b $-2(a - 6)(1 + 2a) = -2(a + 2a^2 - 6 - 12a) = -2(-11a + 2a^2 - 6) = 22a - 4a^2 + 12$

c $(2n)^2 + 2(3n - 1)(2 - n) = 4n^2 + 2(6n - 3n^2 - 2 + n) = 4n^2 + 12n - 6n^2 - 4 + 2n = -2n^2 + 14n - 4$

d $(3p)^2 - 4p(2p - 1) = 9p^2 - 8p^2 + 4p = p^2 + 4p$

2 **a** $4(x + 1)^2 = 4(x^2 + 2x + 1) = 4x^2 + 8x + 4$

b $-3(2p - 5)^2 = -3(4p^2 - 20p + 25) = -12p^2 + 60p - 75$

c $-2(3a + 1)(3a - 1) = -2(9a^2 - 1) = -18a^2 + 2$

d $-2(n + 4)^2 - 4n(n - 2) = -2(n^2 + 8n + 16) - 4n^2 + 8n = -2n^2 - 16n - 32 - 4n^2 + 8n = -6n^2 - 8n - 32$

3 **a** $(2n)^2 - (2 - n)^2 = 4n^2 - (4 - 4n + n^2) = 4n^2 - 4 + 4n - n^2 = 3n^2 - 4 + 4n$

b $(a + 3)^2 - 2(3 - a)^2 = a^2 + 6a + 9 - 2(9 - 6a + a^2) = a^2 + 6a + 9 - 18 + 12a - 2a^2 = -a^2 + 18a - 9$

c $2(x - 1)(2x + 3) - 4(x - 1)^2 = 2(2x^2 + 3x - 2x - 3) - 4(x^2 - 2x + 1) =$
$4x^2 + 6x - 4x - 6 - 4x^2 + 8x - 4 = 10x - 10$

d $-3(2p + 1)^2 - 6(2p + 3)(1 - p) = -3(4p^2 + 4p + 1) - 6(2p - 2p^2 + 3 - 3p) =$
$-12p^2 - 12p - 3 - 12p + 12p^2 - 18 + 18p = -6p - 21$

4 **a** $\frac{15x}{25xy} = \frac{3}{5y}$

b $\frac{4x}{6} = \frac{2x}{3} = \frac{2}{3}x$

c $\frac{ab}{3a} = \frac{b}{3} = \frac{1}{3}b$

d $\frac{24pq}{30pr} = \frac{4q}{5r}$

e $\frac{13km}{13klm} = \frac{1}{l}$

f $\frac{9abc}{15bc} = \frac{3a}{5} = \frac{3}{5}a$

5 **a** $\frac{2xy}{4y} + 5x = \frac{x}{2} + 5x = \frac{1}{2}x + 5x = 5\frac{1}{2}x$

b $\frac{20pq}{5q} - \frac{3pr}{2r} = 4p - \frac{3p}{2} = 4p - \frac{3}{2}p = 4p - 1\frac{1}{2}p = 2\frac{1}{2}p$

c $\frac{6ab}{3a} + \frac{8bc}{12c} = 2b + \frac{2b}{3} = 2b + \frac{2}{3}b = 2\frac{2}{3}b$

6 **a** $\frac{2ab}{2a^2 + 8a} = \frac{2ab}{2a(a+4)} = \frac{b}{a+4}$

b $\frac{3x^2 - 6x}{12x} = \frac{3x(x-2)}{12x} = \frac{x-2}{4} = \frac{1}{4}(x-2) = \frac{1}{4}x - \frac{1}{2}$

c $\frac{n^2 + 3n}{n^2 + 5n + 6} = \frac{n(n+3)}{(n+2)(n+3)} = \frac{n}{n+2}$

d $\frac{p^2 - 4p - 5}{p^2 - 25} = \frac{(p+1)(p-5)}{(p+5)(p-5)} = \frac{p+1}{p+5}$

7 **a** $\frac{1}{2x} + \frac{5}{3x} = \frac{3}{6x} + \frac{10}{6x} = \frac{13}{6x}$

b $\frac{5}{3a} - \frac{2}{b} = \frac{5b}{3ab} - \frac{6a}{3ab} = \frac{5b - 6a}{3ab}$

c $3 + \frac{5}{2p} = \frac{3}{1} + \frac{5}{2p} = \frac{6p}{2p} + \frac{5}{2p} = \frac{6p + 5}{2p}$

Bladzijde 195

8 **a** $\frac{2}{x} \cdot \frac{x}{4y} = \frac{2x}{4xy} = \frac{1}{2y}$

b $\frac{5}{2a} : \frac{b}{4a} = \frac{5}{2a} \cdot \frac{4a}{b} = \frac{20a}{2ab} = \frac{10}{b}$

c $\frac{\left(\frac{2}{x}\right)}{5} = \frac{1}{5} \cdot \frac{2}{x} = \frac{2}{5x}$

d $\frac{4}{\left(\frac{3}{a}\right)} = 4 \cdot \frac{a}{3} = \frac{4a}{3} = \frac{4}{3}a = 1\frac{1}{3}a$

e $\frac{\left(\frac{9}{2a}\right)}{3} = \frac{1}{3} \cdot \frac{9}{2a} = \frac{9}{6a} = \frac{3}{2a}$

f $\frac{10}{\left(\frac{5}{2p}\right)} = 10 \cdot \frac{2p}{5} = \frac{20p}{5} = 4p$

9 **a** $\frac{3a}{5}+\frac{4a-1}{2}=\frac{6a}{10}+\frac{5(4a-1)}{10}=\frac{6a+20a-5}{10}=\frac{26a-5}{10}$

b $\frac{x-1}{3}-\frac{x+3}{2}=\frac{2(x-1)}{6}-\frac{3(x+3)}{6}=\frac{2(x-1)-3(x+3)}{6}=\frac{2x-2-3x-9}{6}=\frac{-x-11}{6}$

c $1\frac{1}{2}+\frac{a-4}{a}=\frac{3}{2}+\frac{a-4}{a}=\frac{3a}{2a}+\frac{2(a-4)}{2a}=\frac{3a+2a-8}{2a}=\frac{5a-8}{2a}$

10 **a** $x^2\cdot x^5+2x^3\cdot 5x^4=x^7+10x^7=11x^7$

b $2x^2\cdot -3x^3+4x^4\cdot x=-6x^5+4x^5=-2x^5$

c $3x^2\cdot y^5\cdot -2x^7\cdot 4x^4=-24x^{13}y^5$

d $\frac{20x^{20}}{5x^5}=4x^{15}$

e $x^4\cdot 5y^3-2y^3\cdot -3x^4=5x^4y^3+6x^4y^3=11x^4y^3$

f $(3p^2)^3+3(p^3)^2=27p^6+3p^6=30p^6$

g $(-2a^5)^3\cdot 3(a^2)^4=-8a^{15}\cdot 3a^8=-24a^{23}$

h $\frac{15x^8y^3}{3x^3y^3}=5x^5$

11 **a** $4x^2(x^2+2x)-(2x^2)^2=4x^4+8x^3-4x^4=8x^3$

b $x^3(x^5-3x)-(2x^2)^3\cdot x^2=x^8-3x^4-8x^6\cdot x^2=x^8-3x^4-8x^8=-7x^8-3x^4$

c $\frac{(3x^5)^3\cdot 2x^2}{3(x^3)^4}=\frac{27x^{15}\cdot 2x^2}{3x^{12}}=\frac{54x^{17}}{3x^{12}}=18x^5$

d $-5(a^5)^3\cdot a-(-4a^8)^2+a^7\cdot 3a^9=-5a^{15}\cdot a-16a^{16}+3a^{16}=-5a^{16}-16a^{16}+3a^{16}=-18a^{16}$

e $(5a^3)^4-(-2a^5)^2+a\cdot a^{11}=625a^{12}-4a^{10}+a^{12}=626a^{12}-4a^{10}$

f $\frac{(4x^4)^3-28(x^2)^6}{12(x^3)^4}=\frac{64x^{12}-28x^{12}}{12x^{12}}=\frac{36x^{12}}{12x^{12}}=3$

12 **a** $3\sqrt{48}=3\cdot\sqrt{16}\cdot\sqrt{3}=3\cdot 4\sqrt{3}=12\sqrt{3}$

b $\sqrt{5}\cdot\sqrt{10}=\sqrt{50}=\sqrt{25}\cdot\sqrt{2}=5\sqrt{2}$

c $4\sqrt{12}\cdot 3\sqrt{6}=12\sqrt{72}=12\cdot\sqrt{36}\cdot\sqrt{2}=12\cdot 6\sqrt{2}=72\sqrt{2}$

Bladzijde 196

13 **a** $\frac{12\sqrt{35}}{4\sqrt{7}}=3\sqrt{5}$

b $\frac{\sqrt{20}}{2\sqrt{5}}=\frac{\sqrt{4}\cdot\sqrt{5}}{2\sqrt{5}}=\frac{2\sqrt{5}}{2\sqrt{5}}=1$

c $\frac{15\sqrt{60}}{\sqrt{5}}=15\sqrt{12}=15\cdot\sqrt{4}\cdot\sqrt{3}=15\cdot 2\sqrt{3}=30\sqrt{3}$

14 **a** $\sqrt{5\frac{5}{9}}=\sqrt{\frac{50}{9}}=\frac{\sqrt{50}}{\sqrt{9}}=\frac{\sqrt{25}\cdot\sqrt{2}}{3}=\frac{5\sqrt{2}}{3}=\frac{5}{3}\sqrt{2}=1\frac{2}{3}\sqrt{2}$

b $\sqrt{6\frac{3}{4}}=\sqrt{\frac{27}{4}}=\frac{\sqrt{27}}{\sqrt{4}}=\frac{\sqrt{9}\cdot\sqrt{3}}{2}=\frac{3\sqrt{3}}{2}=\frac{3}{2}\sqrt{3}=1\frac{1}{2}\sqrt{3}$

c $\sqrt{1\frac{13}{36}}=\sqrt{\frac{49}{36}}=\frac{\sqrt{49}}{\sqrt{36}}=\frac{7}{6}=1\frac{1}{6}$

15 **a** $\sqrt{24}+3\sqrt{6}=\sqrt{4}\cdot\sqrt{6}+3\sqrt{6}=2\sqrt{6}+3\sqrt{6}=5\sqrt{6}$

b $3\sqrt{125}-2\sqrt{45}=3\cdot\sqrt{25}\cdot\sqrt{5}-2\cdot\sqrt{9}\cdot\sqrt{5}=3\cdot 5\sqrt{5}-2\cdot 3\sqrt{5}=15\sqrt{5}-6\sqrt{5}=9\sqrt{5}$

c $4\sqrt{27}-2\sqrt{3}\cdot\sqrt{6}=4\cdot\sqrt{9}\cdot\sqrt{3}-2\sqrt{18}=4\cdot 3\sqrt{3}-2\cdot\sqrt{9}\cdot\sqrt{2}=12\sqrt{3}-2\cdot 3\sqrt{2}=12\sqrt{3}-6\sqrt{2}$

16 **a** $\frac{5}{\sqrt{6}} = \frac{5 \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{5\sqrt{6}}{6} = \frac{5}{6}\sqrt{6}$

b $\sqrt{1\frac{1}{2}} = \sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{6}}{2} = \frac{1}{2}\sqrt{6}$

c $\frac{6}{\sqrt{3}} = \frac{6 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

d $\sqrt{11\frac{1}{4}} = \sqrt{\frac{45}{4}} = \frac{\sqrt{45}}{\sqrt{4}} = \frac{\sqrt{9} \cdot \sqrt{5}}{2} = \frac{3\sqrt{5}}{2} = \frac{3}{2}\sqrt{5} = 1\frac{1}{2}\sqrt{5}$

e $\frac{7}{\sqrt{7}} + \sqrt{\frac{1}{7}} = \frac{7 \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}} + \frac{\sqrt{1}}{\sqrt{7}} = \frac{7\sqrt{7}}{7} + \frac{1 \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}} = \sqrt{7} + \frac{\sqrt{7}}{7} = \sqrt{7} + \frac{1}{7}\sqrt{7} = 1\frac{1}{7}\sqrt{7}$

f $\frac{2}{\sqrt{8}} - \sqrt{4\frac{1}{2}} = \frac{2}{\sqrt{4} \cdot \sqrt{2}} - \sqrt{\frac{9}{2}} = \frac{2}{2\sqrt{2}} - \frac{\sqrt{9}}{\sqrt{2}} = \frac{1}{\sqrt{2}} - \frac{3}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\frac{2 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$

17 **a** $3x + y = 7$
$y = -3x + 7$
Dus $rc_l = -3$.

b $x - 2y = 8$
$-2y = -x + 8$
$y = \frac{1}{2}x - 4$
Dus $rc_m = \frac{1}{2}$.

c $2x - 5y = 10$
$-5y = -2x + 10$
$y = \frac{2}{5}x - 2$
Dus $rc_n = \frac{2}{5}$.

18 **a** $3y$ euro

b $4x + 3y$ euro

c $4x + 3y = 150$

d $x = 18$ geeft $4 \cdot 18 + 3y = 150$
$72 + 3y = 150$
$3y = 150 - 72$
$3y = 78$
$y = 26$
Dus Petra heeft die dag 26 bakjes bramen verkocht.

Onderzoek Pythagoreïsche drietallen

Bladzijde 197

1 **a** $(m^2 - n^2)^2 = m^4 - 2m^2n^2 + n^4$
$(2mn)^2 = 4m^2n^2$
$(m^2 + n^2)^2 = m^4 + 2m^2n^2 + n^4$

b $(m^2 - n^2)^2 + (2mn)^2 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2 = m^4 + 2m^2n^2 + n^4 = (m^2 + n^2)^2$

c Het product van twee gehele getallen is een geheel getal, m en n zijn positieve gehele getallen, dus:
- $m^2 = m \cdot m$ is een positief geheel getal
- mn is een positief geheel getal, en dus is $2mn$ ook een positief geheel getal
- $n^2 = n \cdot n$ is een positief geheel getal.

De som van twee gehele getallen is een geheel getal.
m^2 is een positief geheel getal en n^2 is een positief geheel getal, dus:
- $m^2 + n^2$ is een positief geheel getal
- $m^2 - n^2 = m^2 + -n^2$ is een positief geheel getal voor $m > n$.

Dus $m^2 - n^2$, $2mn$ en $m^2 + n^2$ zijn positieve gehele getallen.
En omdat $(m^2 - n^2)^2 + (2mn)^2 = (m^2 + n^2)^2$ is $[m^2 - n^2, 2mn, m^2 + n^2]$ een Pythagoreïsch drietal voor $m > n$.

d $m = 4$ en $n = 2$ geeft het drietal $[4^2 - 2^2, 2 \cdot 4 \cdot 2, 4^2 + 2^2] = [12, 16, 20]$.

e $m = 2$ en $n = 1$ geeft het drietal $[2^2 - 1^2, 2 \cdot 2 \cdot 1, 2^2 + 1^2] = [3, 4, 5]$.
$m = 3$ en $n = 2$ geeft het drietal $[3^2 - 2^2, 2 \cdot 3 \cdot 2, 3^2 + 2^2] = [5, 12, 13]$.

f Er moet gelden $m^2 + n^2 = 11$.
$m = 2$ en $n = 1$ geeft $m^2 + n^2 = 2^2 + 1^2 = 4 + 1 = 5 < 11$
$m = 3$ en $n = 1$ geeft $m^2 + n^2 = 3^2 + 1^2 = 9 + 1 = 10 < 11$
$m = 4$ en $n = 1$ geeft $m^2 + n^2 = 4^2 + 1^2 = 16 + 1 = 17 > 11$
$m = 3$ en $n = 2$ geeft $m^2 + n^2 = 3^2 + 2^2 = 9 + 4 = 13 > 11$
Er bestaat geen Pythagoreïsch drietal waarvan het grootste getal 11 is.

Er moet gelden $2mn = 11$ of $m^2 - n^2 = 11$.
$2mn$ is even, dus $2mn$ kan niet 11 zijn.
Uit $m^2 - n^2 = 11$ ofwel $(m + n)(m - n) = 11$ volgt dat moet gelden $m + n = 11$ en $m - n = 1$, en dit geeft $m = 6$ en $n = 5$.
Dus er bestaat een Pythagoreïsch drietal waarvan het kleinste getal 11 is.
Dat is het drietal $[6^2 - 5^2, 2 \cdot 6 \cdot 5, 6^2 + 5^2] = [11, 60, 61]$.

2 **a**

m	1	2	3	4	5	…
$2m$	2	4	6	8	10	…
$m^2 - 1$	0	3	8	15	24	…

Vanaf $m = 3$ is het getal $2m$ kleiner dan het getal $m^2 - 1$.

b $2m = 30$
$m = 15$
Dit geeft het drietal $[15^2 - 1, 2 \cdot 15, 15^2 + 1] = [224, 30, 226]$.

$m^2 + 1 = 37$
$m^2 = 36$
$m = 6 \vee m = -6$ (kan niet)
$m = 6$ geeft het drietal $[6^2 - 1, 2 \cdot 6, 6^2 + 1] = [35, 12, 37]$.

$m^2 - 1 + m^2 + 1 = 800$
$2m^2 = 800$
$m^2 = 400$
$m = 20 \vee m = -20$ (kan niet)
$m = 20$ geeft het drietal $[20^2 - 1, 2 \cdot 20, 20^2 + 1] = [399, 40, 401]$.

$m^2 - 1 + 2m + m^2 + 1 = 144$
$2m^2 + 2m = 144$
$2m^2 + 2m - 144 = 0$
$m^2 + m - 72 = 0$
$(m - 8)(m + 9) = 0$
$m - 8 = 0 \vee m + 9 = 0$
$m = 8 \vee m = -9$ (kan niet)
$m = 8$ geeft het drietal $[8^2 - 1, 2 \cdot 8, 8^2 + 1] = [63, 16, 65]$.

3 Ga uit van $[m^2 - n^2, 2mn, m^2 + n^2]$.
Door te proberen vind je dat voor $m = 4$ en $n = 3$ geldt $m^2 + n^2 = 25$.
Hierbij hoort het drietal $[4^2 - 3^2, 2 \cdot 4 \cdot 3, 4^2 + 3^2] = [7, 24, 25]$.

Ga uit van $[m^2 - 1, 2m, m^2 + 1]$ en gebruik de tip die in het leerboek naast de opgave staat.
$2m = 31$ geeft geen Pythagoreïsch drietal, maar $2m = 62$ ofwel $m = 31$ geeft het drietal $[31^2 - 1, 2 \cdot 31, 31^2 + 1] = [960, 62, 962]$. Nu is ook $[480, 31, 481]$ een Pythagoreïsch drietal.
Controle: $480^2 + 31^2 = 231\,361$ en $481^2 = 231\,361$, dus het klopt.